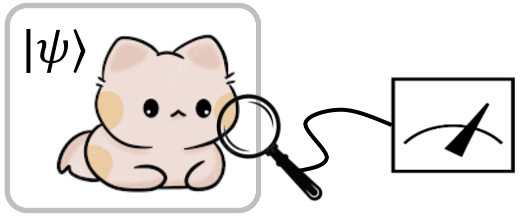


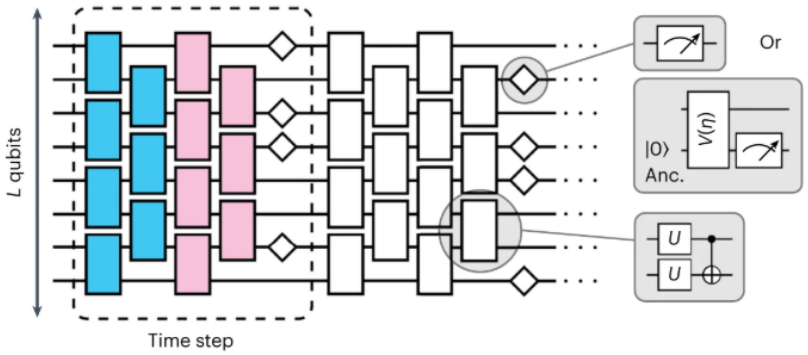

# Error mitigation in quantum dynamics and condensed-matter simulations

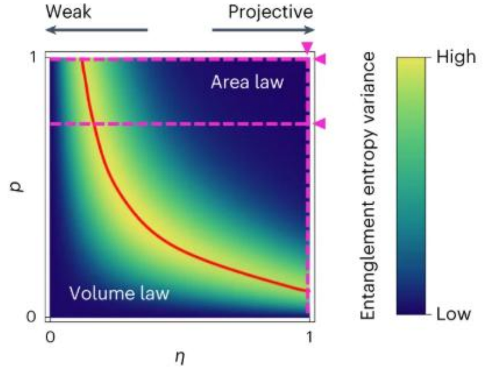
WERQSHOP 2025

**Jin Ming Koh**
Harvard University

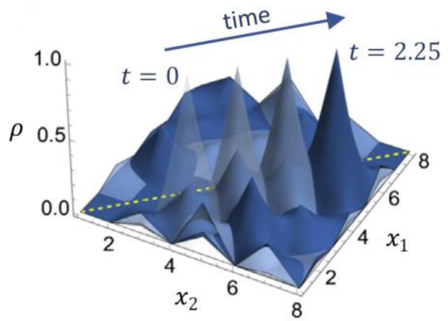
# This talk: Error-mitigated quantum simulation

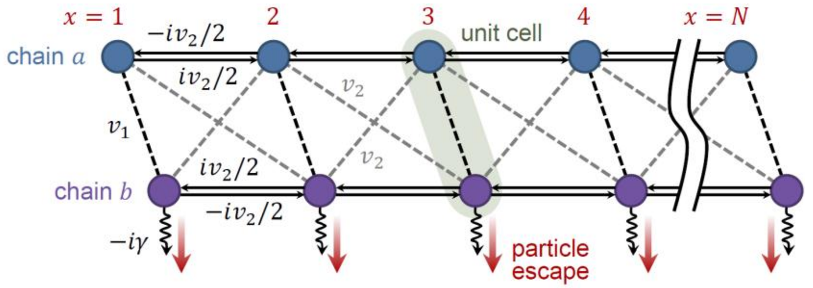
Measurement-induced entanglement phase transitions



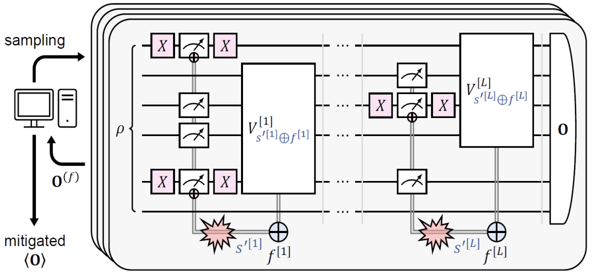



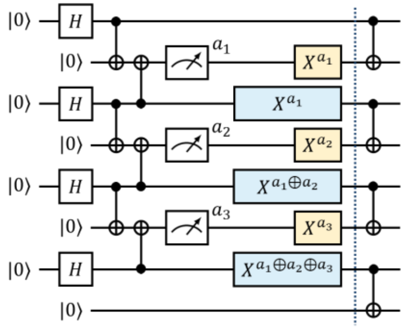

Condensed-matter simulations




Readout error mitigation for dynamic circuits



# Techniques we'll come across

Readout error mitigation for terminal measurements

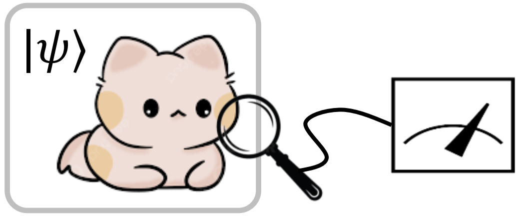
Symmetry verification / post-selection by symmetry sectors

Randomized compilation / gate twirling

Zero noise extrapolation (augmented with physical constraints)

"Ad-hoc" methods: calibration and zeroing of noise contributions

Readout error mitigation for mid-circuit measurements and feedforward




# Part I: Measurement-induced entanglement phase transitions

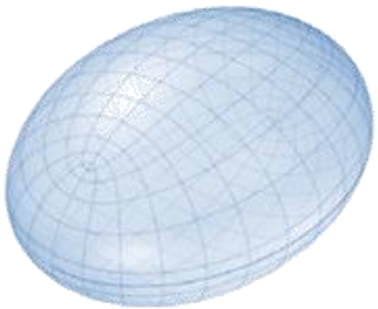
## Measurement-induced entanglement phase transition on a superconducting quantum processor with mid-circuit readout

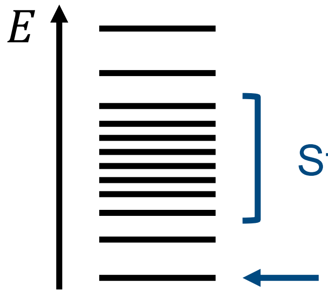
Jin Ming Koh, Shi-Ning Sun, Mario Motta & Austin J. Minnich

*Nature Physics* **19**, 1314–1319 (2023) | Cite this article

# Volume vs. area law

**Volume law:** Amount of entanglement scales with system size — *extensive* entanglement entropy.



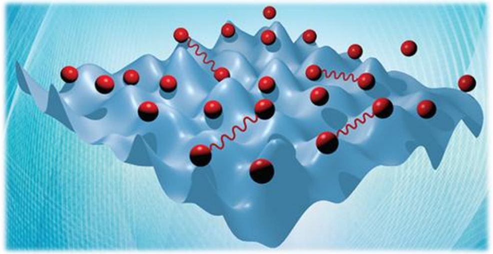



States deep in spectrum (volume law).

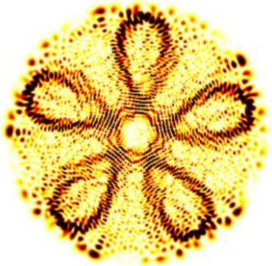
Ground state (area law).

**Area law**: Entanglement scales only with surface area of system — *sub-extensive* entropy.


Topological boundary modes



Many-body localization (MBL)*



Quantum scars*

Image sources: Asbóth *et al.*, A short course on topological insulators, Springer 2016; Lee group website, NUS; Keski-Rahkonen *et al.*, *PRB* **96**, 094204, 2017.

*Up to logarithmic correction factors.

# Unitaries and measurements

Effect of Measurements (Qualitative)

*No measurements*. Many-body quantum systems become increasingly entangled; **volume-law** scaling.

Calabrese & Hardy (2005).
Kim & Huse (2013).
Liu & Suh (2013).
Kaufman *et al.* (2016).
Keyserlingk *et al.* (2018).

*Unitary evolution interspersed with measurements*. Distinct volume- and area-law phases possible.

Entanglement MIPT

Chan *et al.* (2019).
Li, Chen & Fisher (2019).
Skinner, Ruhman & Nahum (2019).
Szyniszewski, Romito & Schomerus (2019).
Zabalo *et al.* (2020).
Nahum *et al.* (2021).

*Quantum Zeno effect*. Locked in measurement subspace.

Misra & Sudarshan (1977).
Wheeler & Zurek (1983).
Zhu *et al.* (2011).

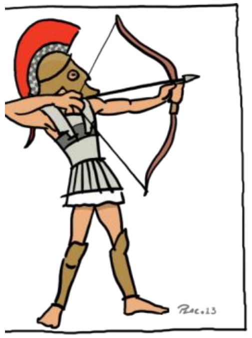
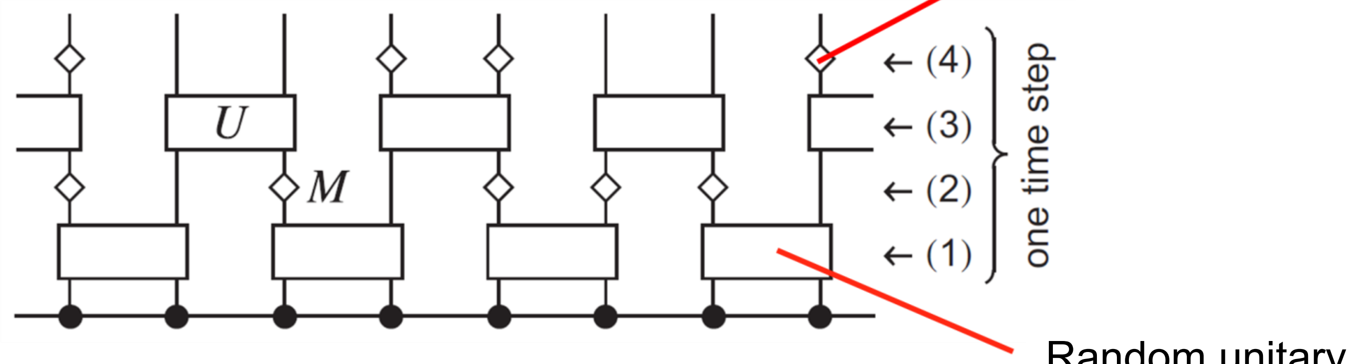


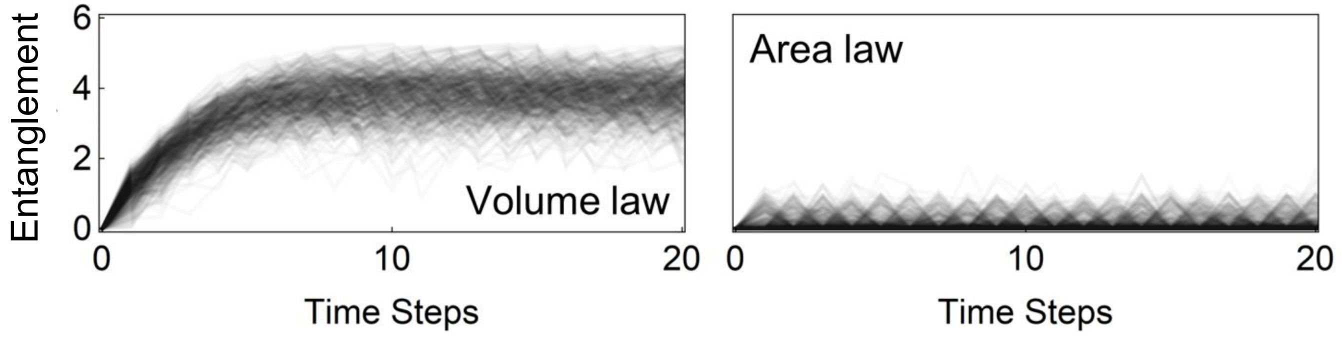
Image sources: https://www.pngwing.com/en/free-png-mfasv

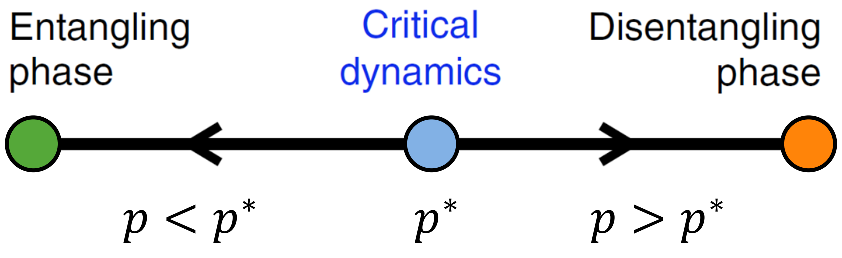
# Entanglement phase transitions

Circuit model: Szyniszewski, et. al. PRB 100(6) 064204 (2019).




Probability $p$ of placing a measurement



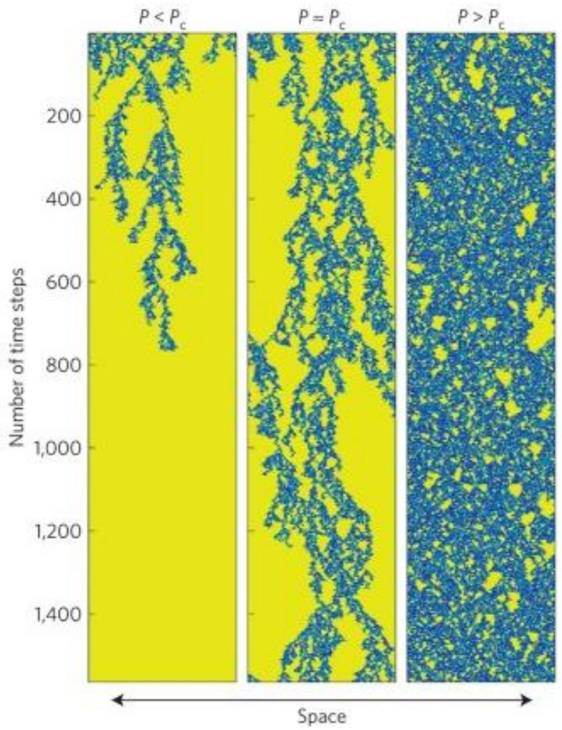




*Roughly:*

Unitaries **generate** entanglement.
Measurements **destroy** entanglement.

# Many connections

**Percolation** phase transitions

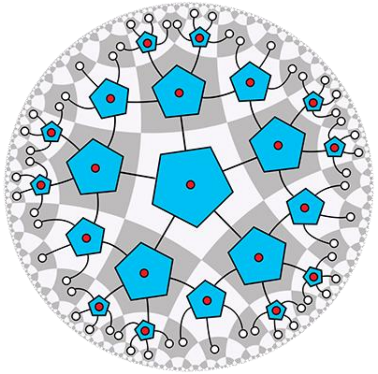



*Semi-classical limit*

Entanglement MIPT

*Exact w/ restrictions*

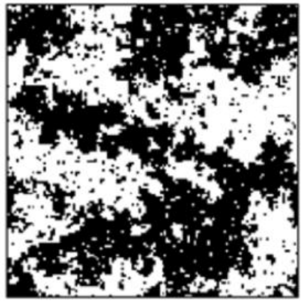
**Quantum error correction**



*Approx.*

(Other) statistical mechanics

*e.g. Random-bond Ising model*



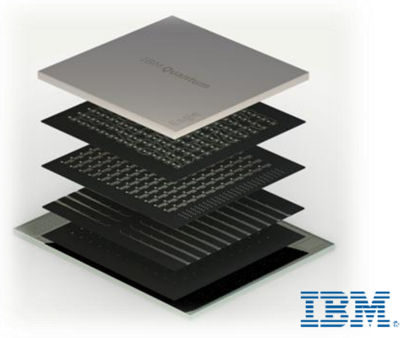
Choi *et al.* PRL 125(3), 030505 (2020)
Gullans *et al.* PRX 10(4), 041020 (2020)
Gullans *et al.* PRL 125(7), 070606 (2020)
Tang *et al.* PRR 2(1), 013022 (2020)
Bao *et al.* PRB 101(10), 104301 (2020)

Image sources: Lemoult *et al.*, *Nat. Phys.* **12**, 254 (2016); Shin *et al.*, *Nat. Comms.* **8**, 1270 (2017); Mazurek *et al.*, PRA **101**, 042305 (2020).

# The question circa ~2021

Can we **physically access** MIPTs?
*(and if so, what can we learn?)*

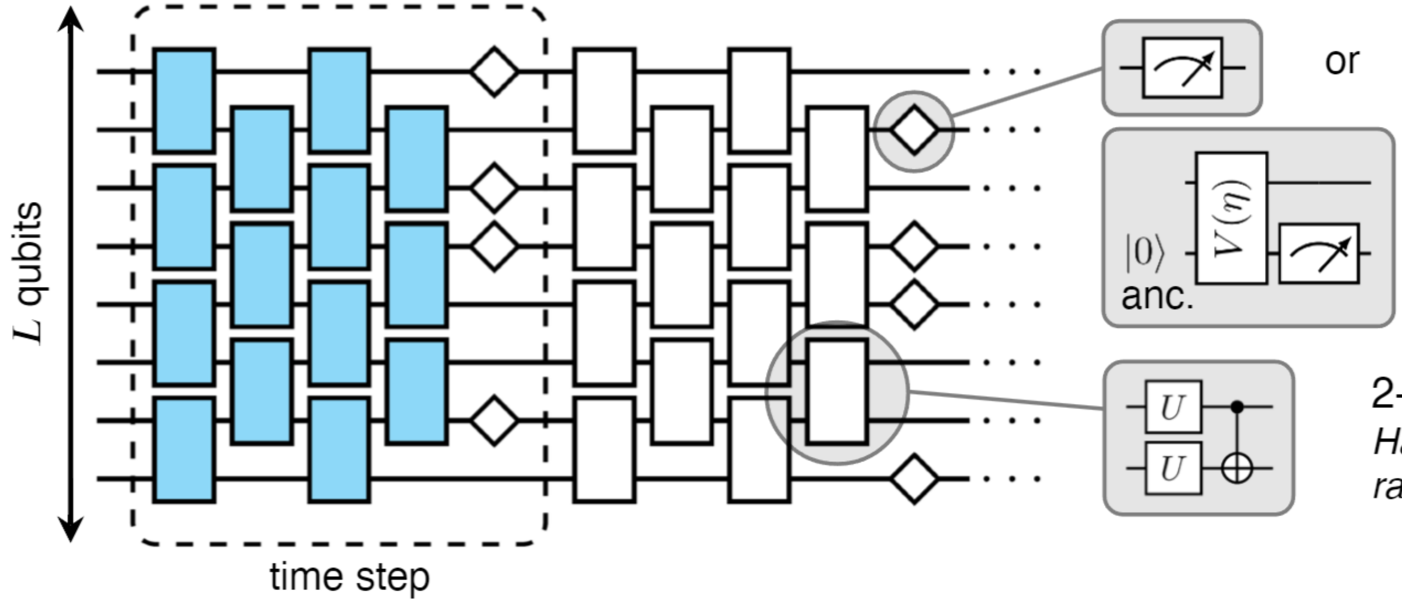
Interspersed measurements…
Control of evolving unitaries over many qubits…
Coherence over sufficiently long times…
Extracting entanglement properties…




**Mid-circuit measurements!**
Sub-microsecond readout ($5\ \mu s \rightarrow \sim 750\ \mathrm{ns}$)


Image sources: IBM Research.

# “Hybrid” random circuits



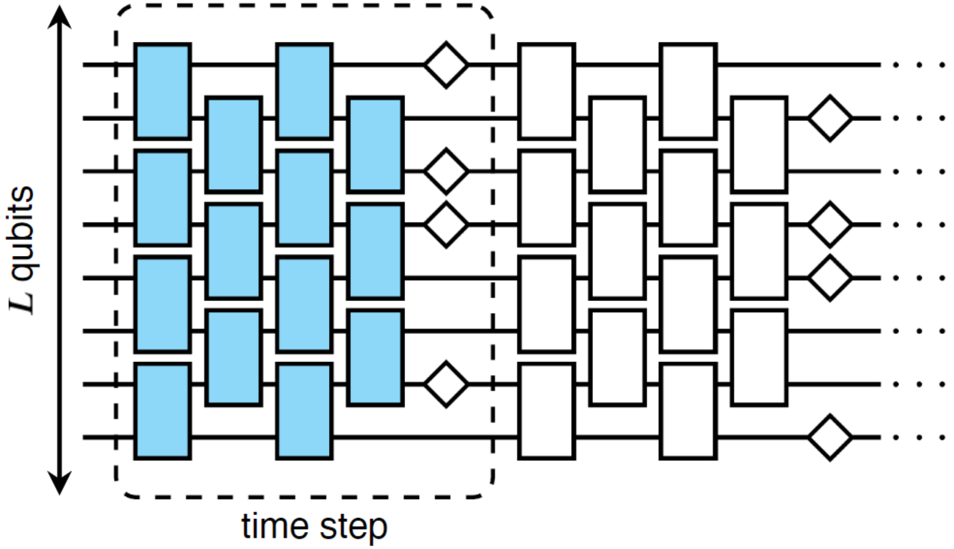

Projective / weak mid-circuit measurements

*Placed on each qubit with probability p*

2-qubit gates

*Haar-random single-qubit rotations + randomly-directed CX*

# Measuring entanglement entropy



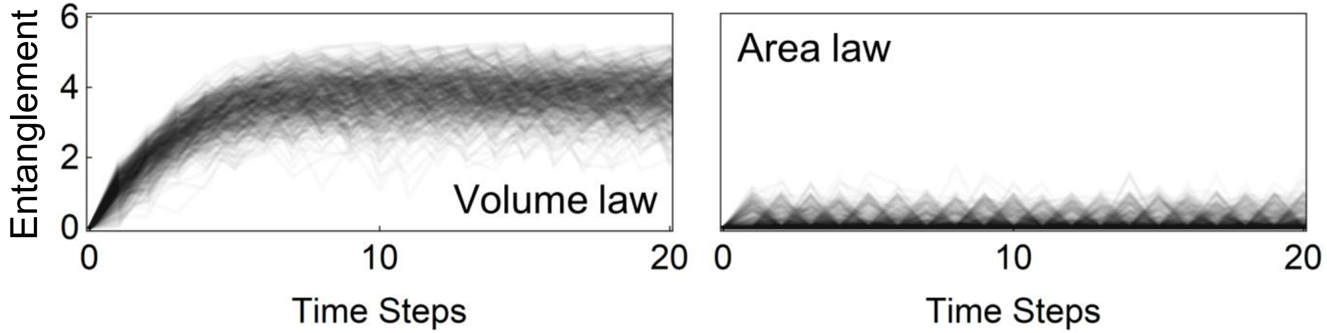

*Tomography (on subsystem)* → Reconstructed quantum state $\rho$ → Entanglement entropy $S$

*Accelerated using simultaneous MUB measurements…*

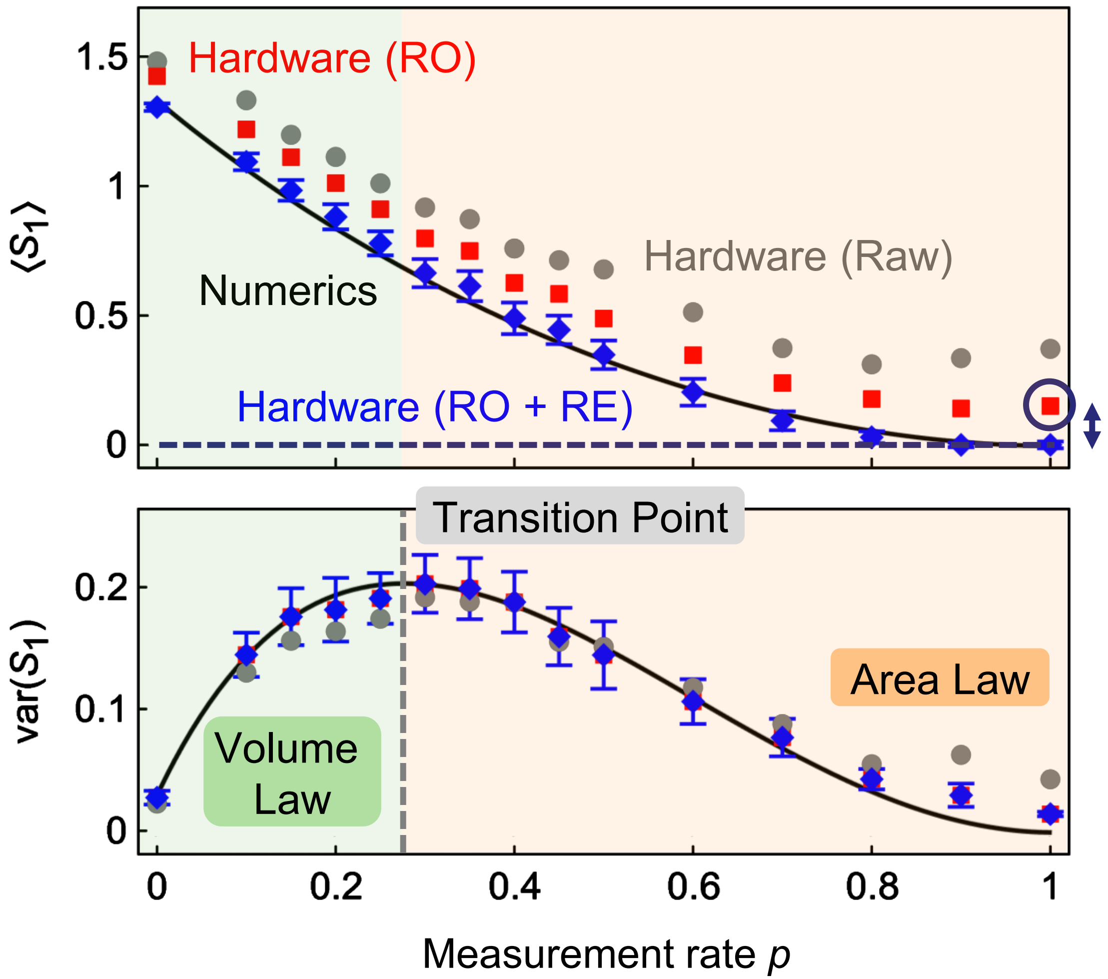
Sample over quantum trajectories:

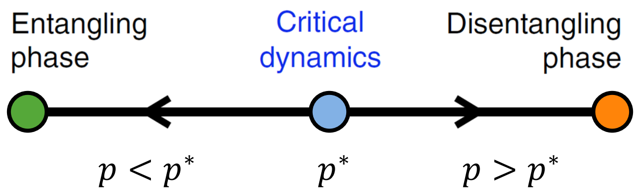
| | | |
|---|---|---|
| 01010010101 | $\rho$ | $S$ |
| 01010000001 | $\rho$ | $S$ |
| 00000010000 | $\rho$ | $S$ |
| ⋮ | ⋮ | ⋮ |
| 01110010000 | $\rho$ | $S$ |

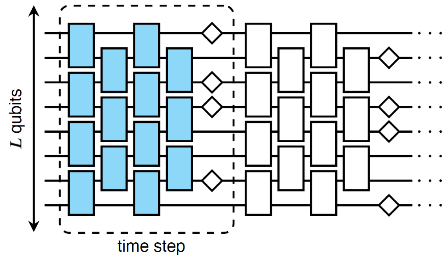
Compute entropy mean, variance.



# Transition w/ projective measurements



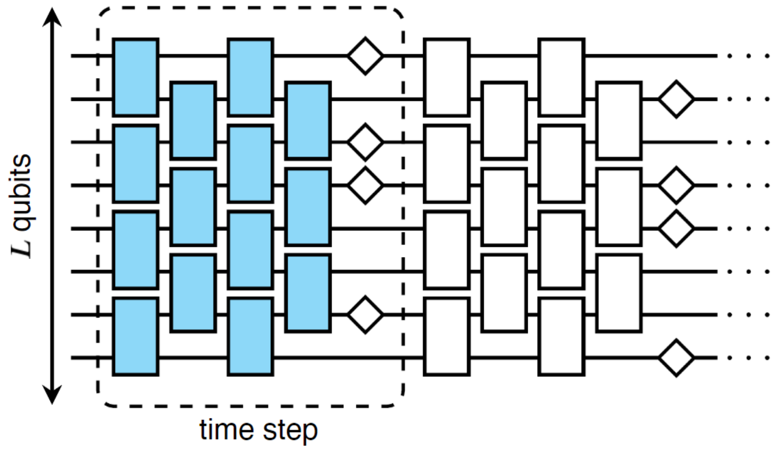




RO – Readout error mitigation.

RE – Residual entropy correction.

**Takeaway**: Observation of entanglement phase transition signature!

# Standard readout error mitigation



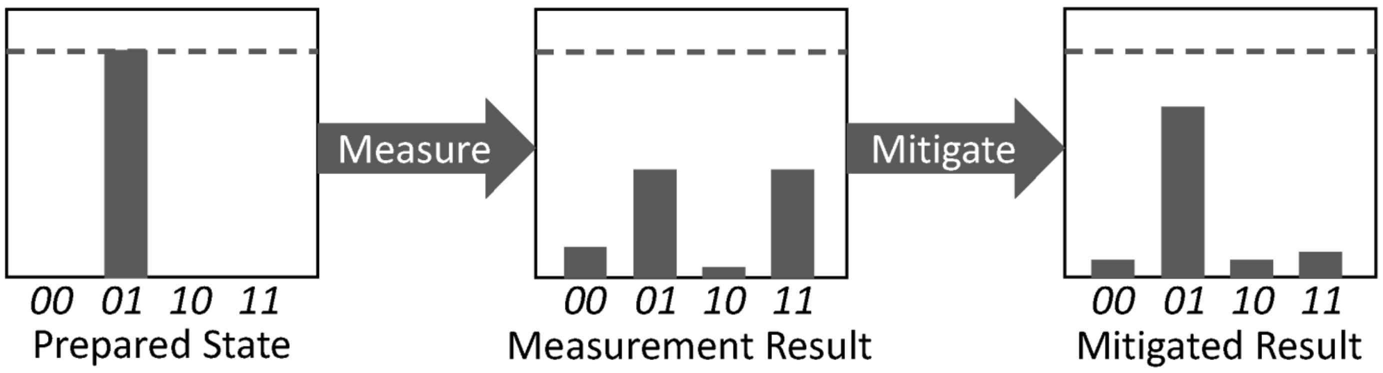
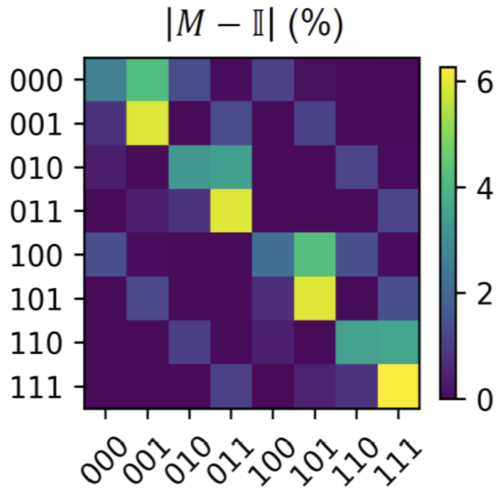




Figure: M. Beisel *et al.*, Configurable Readout Error Mitigation in Quantum Workflows. *Electronics* **2022**, *11*, 2983

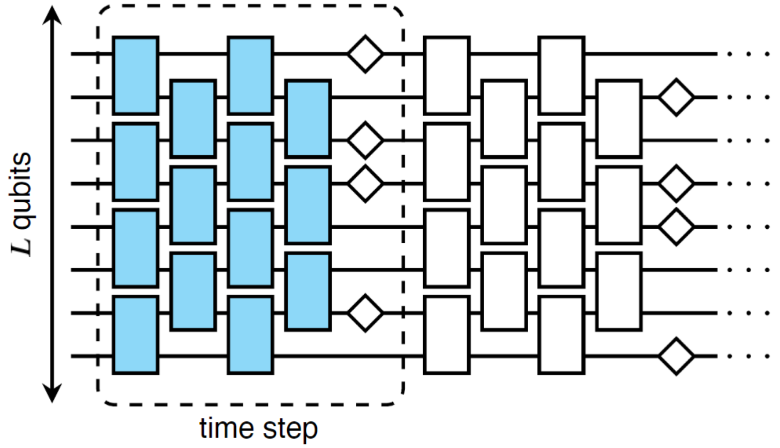



$$\mathbf{p}_{\text{obs}} = M\mathbf{p}_{\text{ideal}} \Longrightarrow \mathbf{p}_{\text{mit}} = M^{+}\mathbf{p}_{\text{obs}}$$

Possible improvements:

- $\mathbf{p}_{\text{mit}}$ may not be $\geq 0$; find closest proper probability distribution (in a suitable norm).
- Knowledge of bitstrings that should not show up.

# Entropy correction ("zeroing")




*Tomography (on subsystem)*

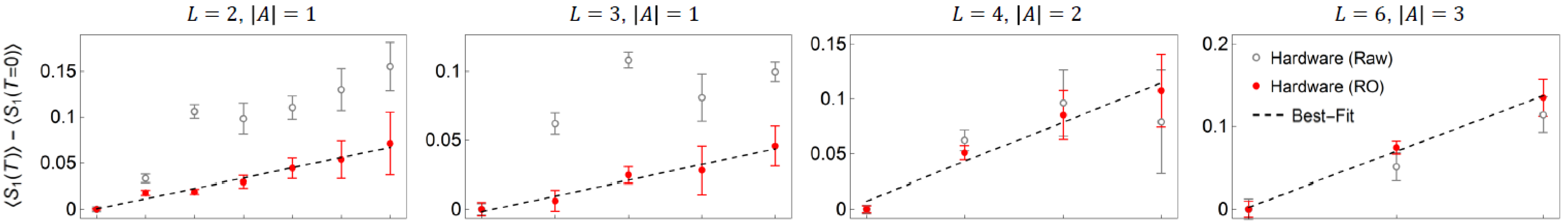
Zeroed (subtracted away) linear entropy contribution:

$$\delta S_\alpha(p,\eta) = \frac{\langle \mathcal{E}[\mathcal{C}_{p,\eta}]\rangle}{\langle \mathcal{E}[\mathcal{C}_{p=\eta=1}]\rangle} S_\alpha(p=\eta=1)$$

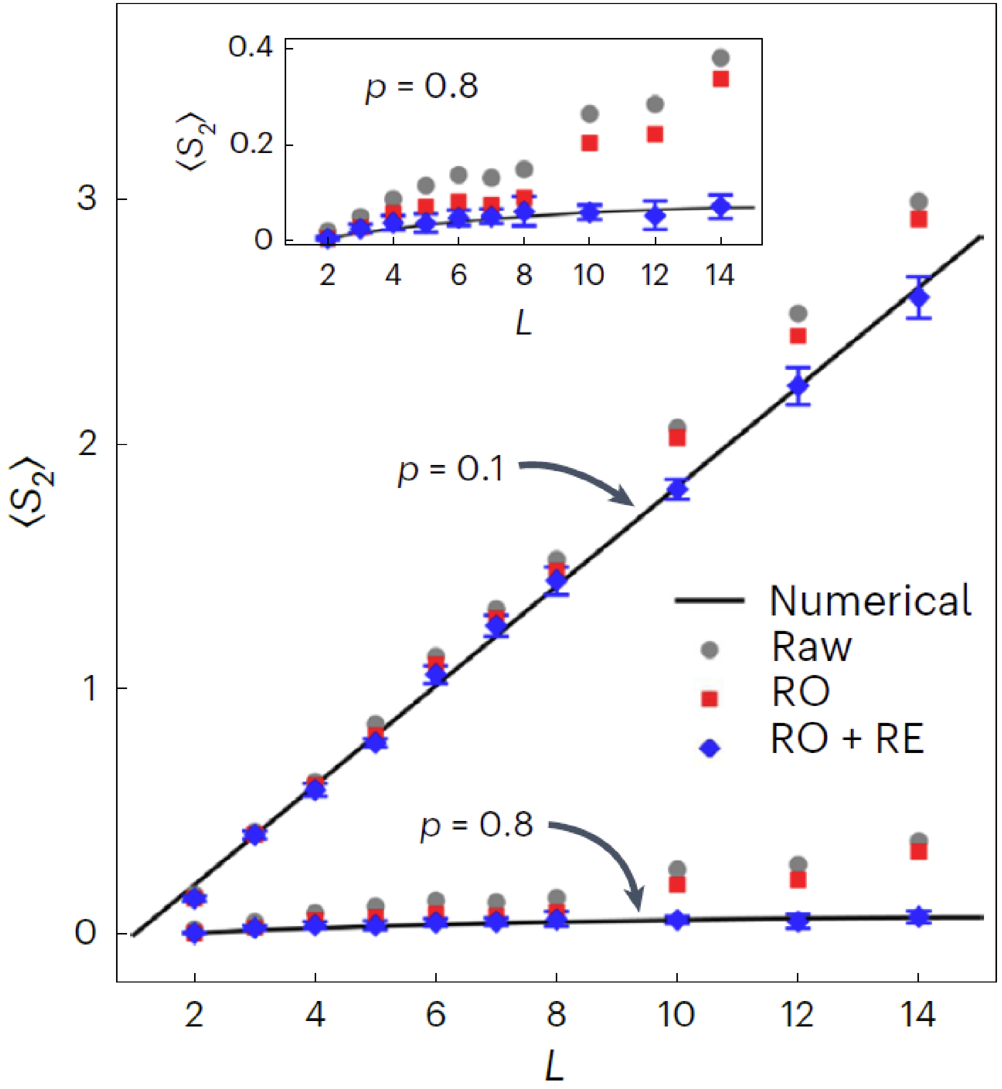
How did we know this is suitable?

Motivated by simple theoretical model, suggests:

$$\langle \delta S_\alpha(p,\eta,T)\rangle - \langle \delta S_\alpha(p,\eta,T=0)\rangle \propto T,$$
$$\langle S_\alpha(T)\rangle - \langle S_\alpha(T=0)\rangle \propto T$$



# *Results* – Scaling w/ system size




**Takeaway:** Direct evidence of volume- and area-law entanglement phases realized on hardware!

# *Results* – Critical behaviour

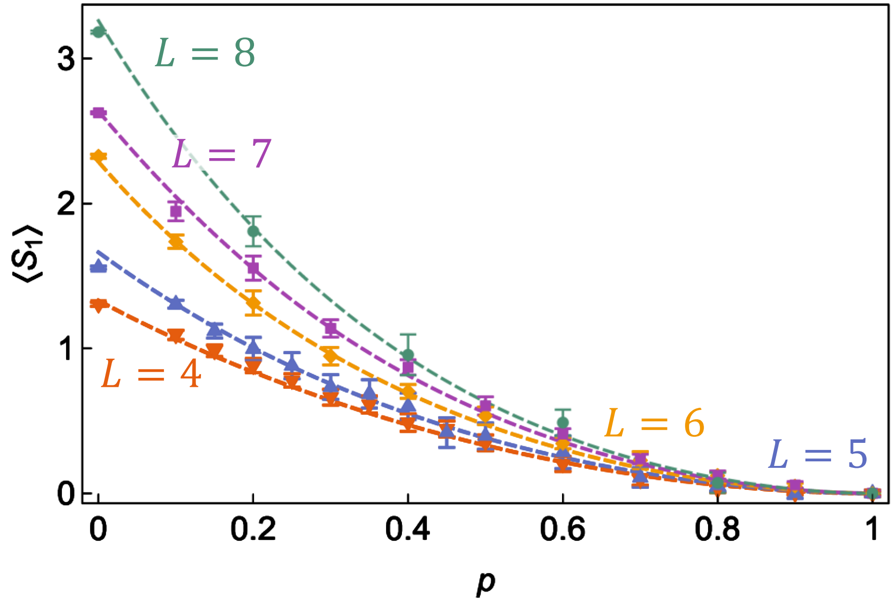
**Scaling near critical point:**

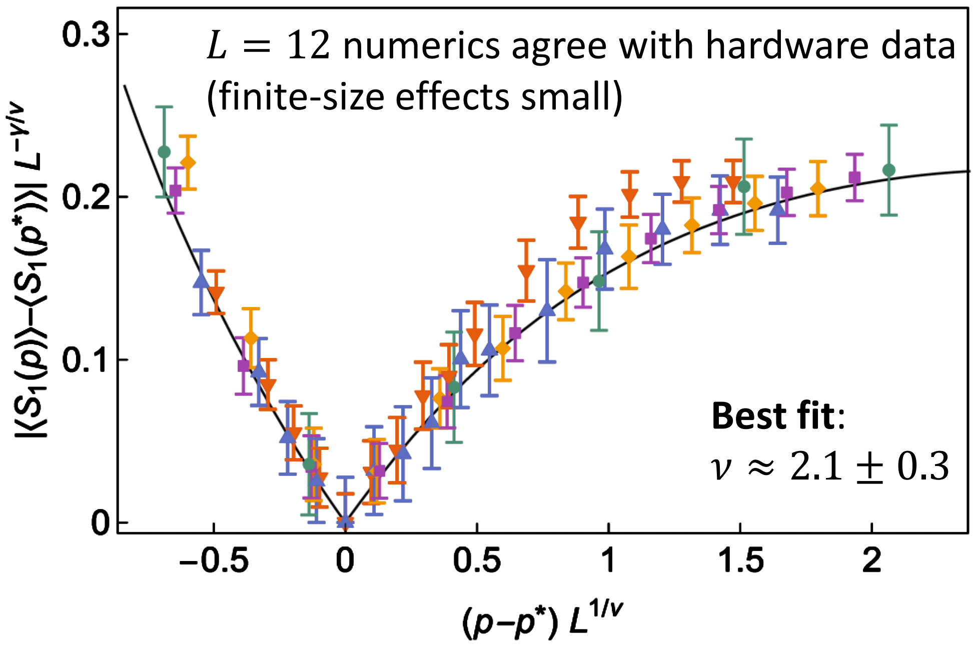
Entanglement entropy $\langle S \rangle \sim |p - p^*|^\gamma$

Correlation length $\xi \sim |p - p^*|^{-\nu}$

$$\langle S \rangle L^{-\gamma/\nu} = F\left[L^{1/\nu}(p - p^*)\right]$$

Rescaled data at all $L$ should collapse onto same curve ($F$) if critical phase transition occurs.




*Prior numerics:*

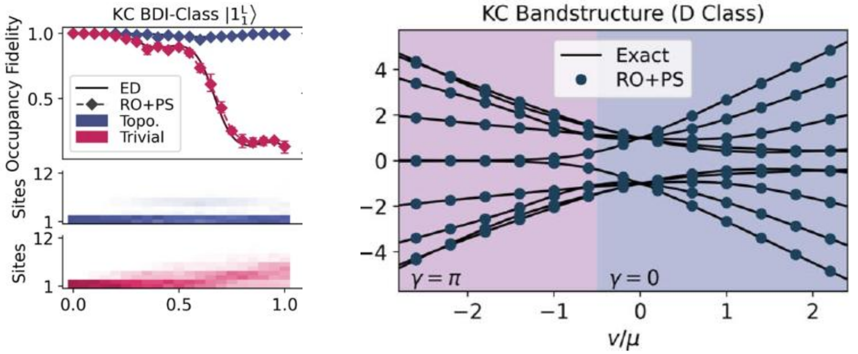
$\nu \approx 2.0 \pm 0.1$ (Szyniszewski *et al.*, *PRB*)

$\nu \approx 2.352 \pm 0.005$ (Skinner *et al.*, *PRX*)

**Takeaway:** Demonstration of phase transition **criticality** from hardware data!

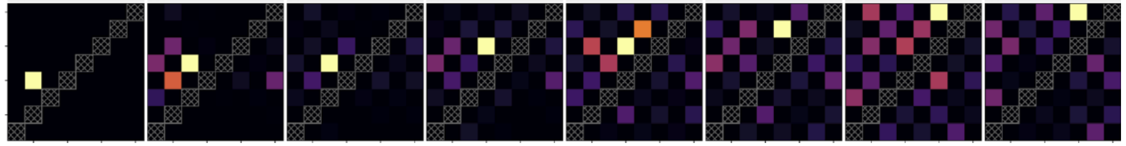
# Part II: Condensed-matter simulations

# 1D, 2D, 3D+ local Hamiltonian simulations



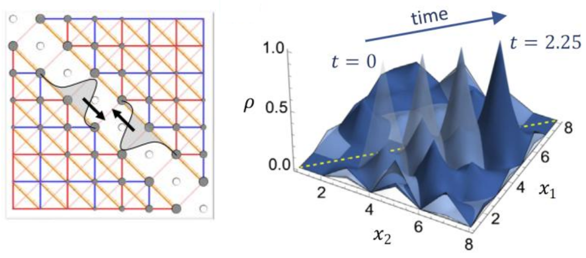

**1D symmetry-protected topological fermion chains + interactions**

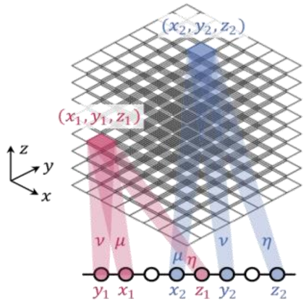
**JMK,** T Tai, YH Phee, WE Ng, CH Lee, *npj QI* (2022).

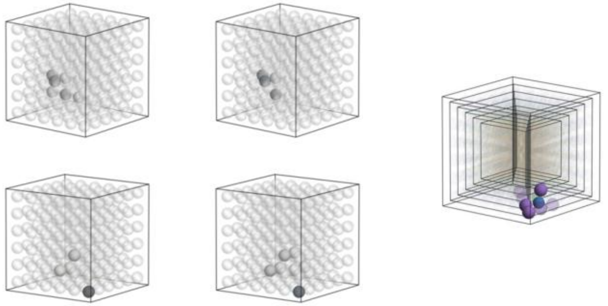


**Interaction-induced chiral topological dynamics on Chern insulator models**

**JMK,** T Tai, CH Lee, *PRL* (2022).

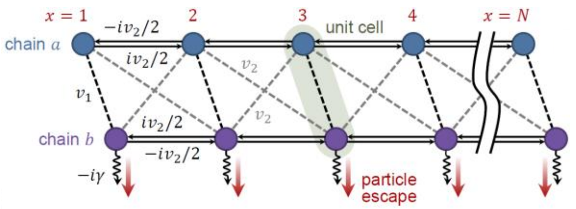


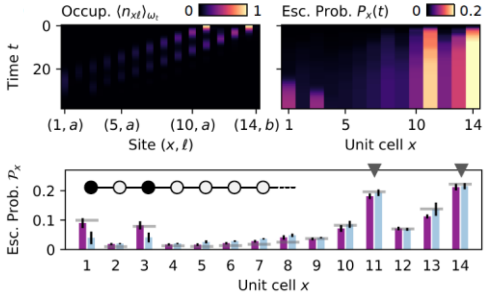



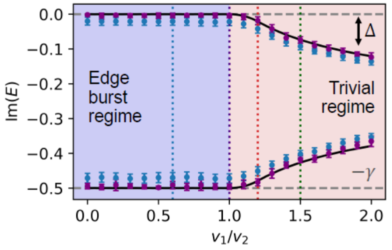



**Higher-order topological lattices in 2-4D mapped onto interacting lower-D models**

**JMK,** T Tai, CH Lee, *Nature Comms* (2024).




**Interacting non-Hermitian edge and cluster bursts**

**JMK,** W-T Xue, T Tai, DE Koh, CH Lee, *arXiv:2503.14595.*

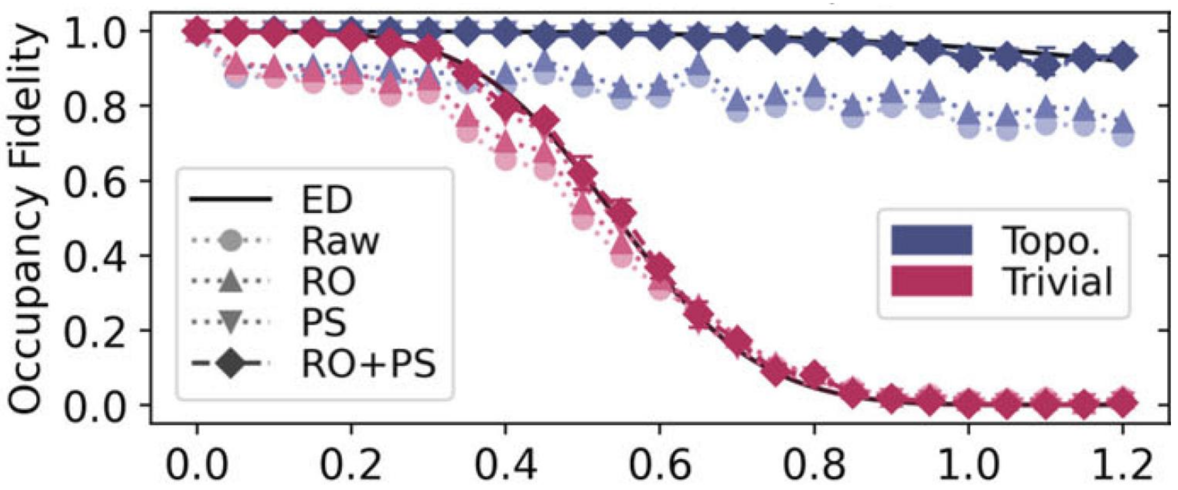
# Post-selection in symmetry sectors

In condensed-matter and chemistry, often interested in simulating in a particular Fock-space sector (say $p$ particles).

Hamiltonian is $U(1)$ number conserving.

Two choices of encoding system states into qubit states:

- “First quantization”: Qubit states associated with $p$-particle wavefunctions [need only $\sim \log_2(n \text{ choose } p)$ qubits].
- “Second quantization”: Qubit states representing entire Fock space; $p$-particle states active during an ideal simulation.

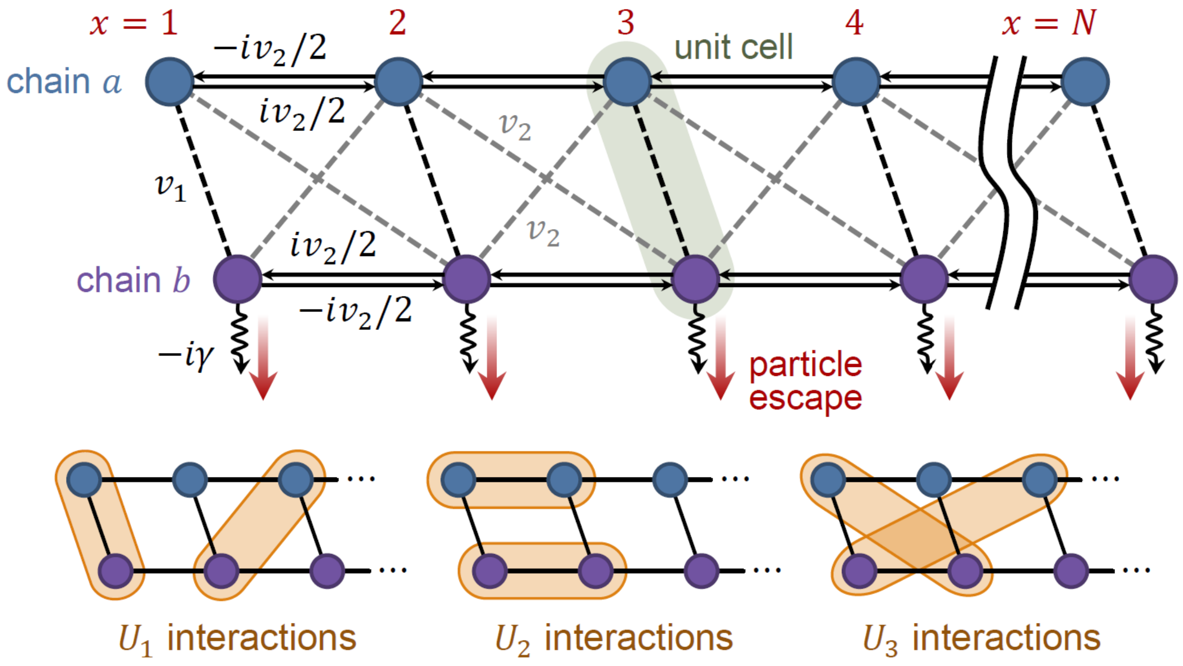


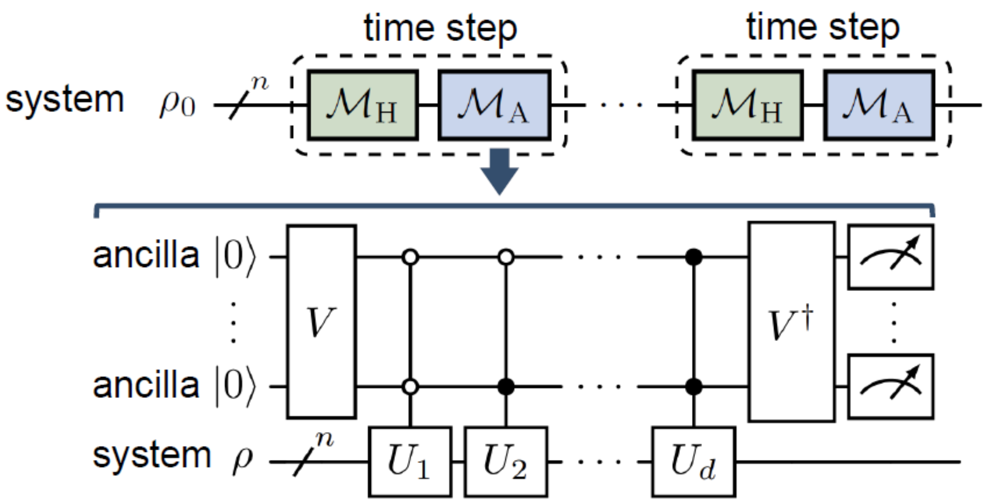

**JMK,** T Tai, YH Phee, WE Ng, CH Lee, *npj QI* (2022).

*Qubit-efficient, but compilation of circuits tricky.*

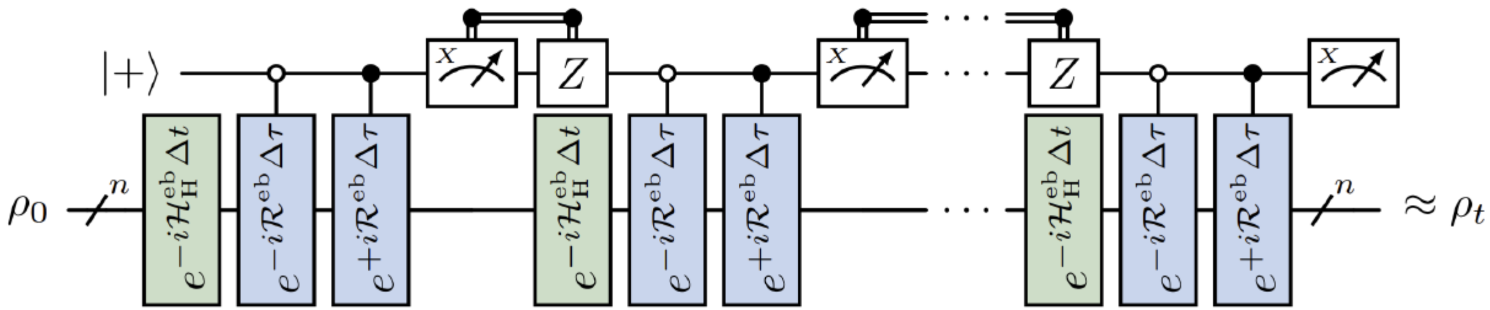
*Measure particle number simultaneously with observable (when possible); post-select shots with correct particle number.*

# Non-Hermitian Hamiltonian simulation



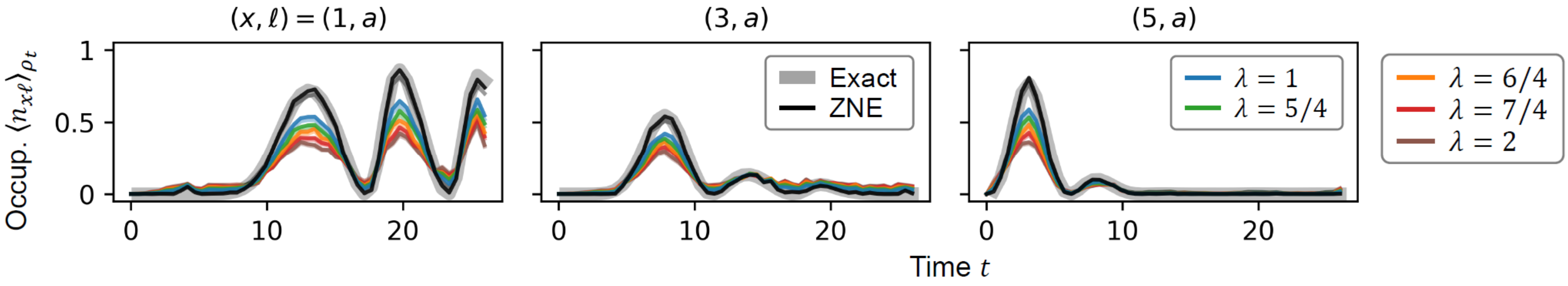

**JMK,** W-T Xue, T Tai, DE Koh, CH Lee, *arXiv:2503.14595.*

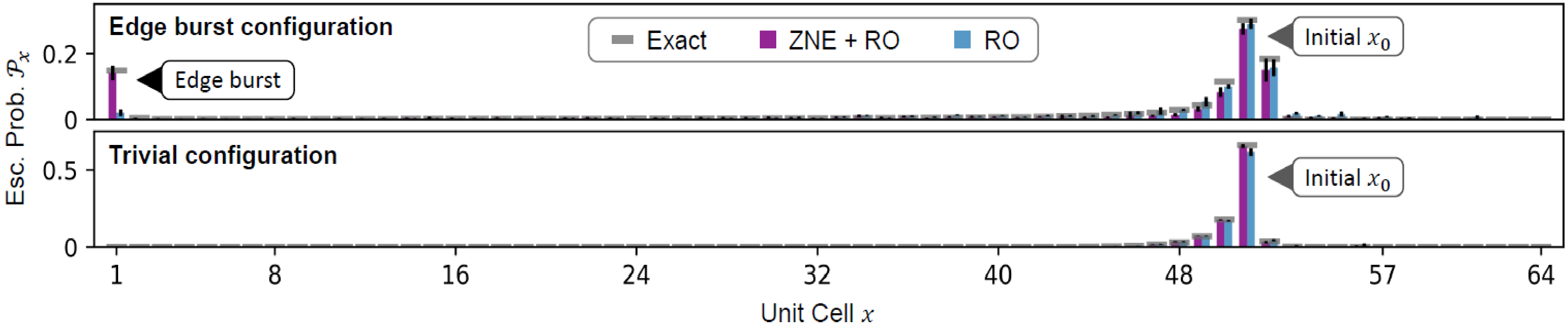


# Pauli twirling + zero noise extrapolation

Additional constraints on ZNE: Must satisfy symmetry or physicality consistency conditions at **all** $\lambda$.

*e.g. fermion occupation, total particle number, (imaginary) energy*



# Part III: Readout error mitigation for dynamic circuits

**Readout Error Mitigation for Mid-Circuit Measurements and Feedforward**

Jin Ming Koh,[1,2] Dax Enshan Koh,[2] and Jayne Thompson[2]

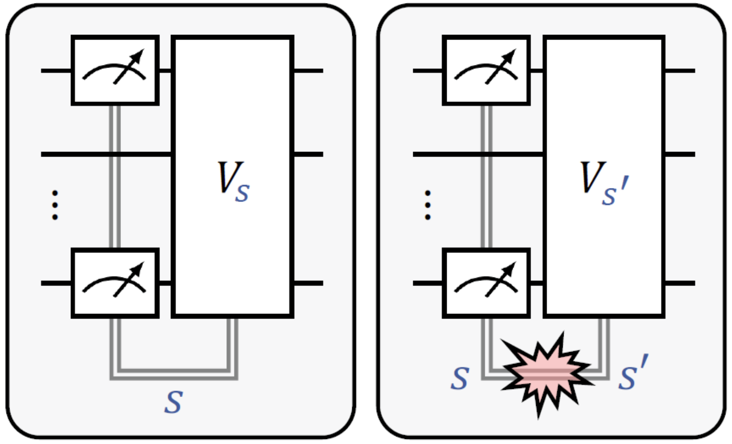
[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *A*STAR Quantum Innovation Centre (Q.InC), Institute of High Performance Computing (IHPC), Agency for Science, Technology and Research (A*STAR), 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632, Republic of Singapore*

**arXiv:2406.07611**

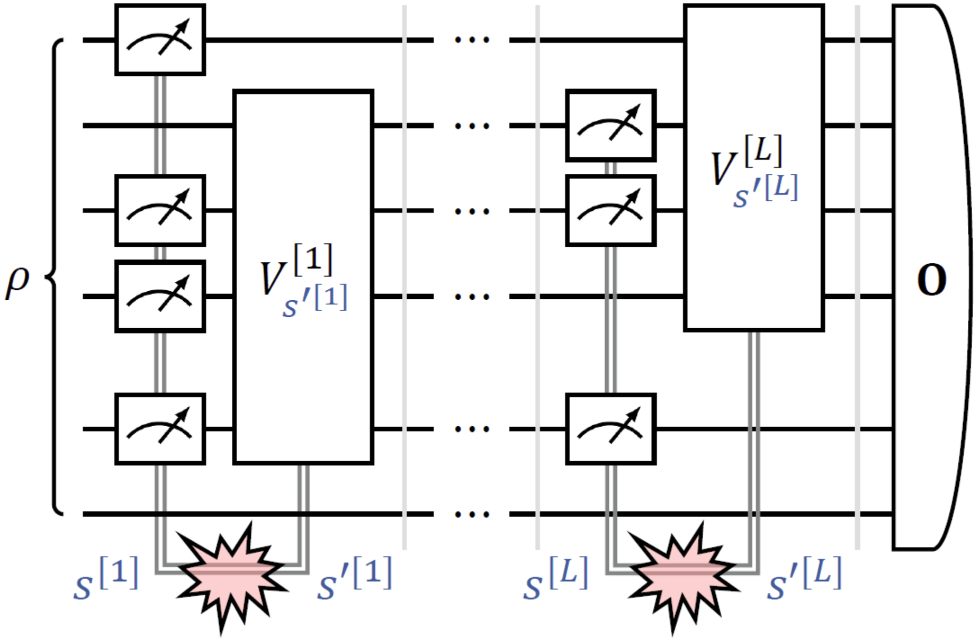
# Readout errors on MCMs

**Incorrect operations** applied during feedforward due to readout errors!

*Think: An if-else branching error in a classical program!*




Ideal    With readout noise



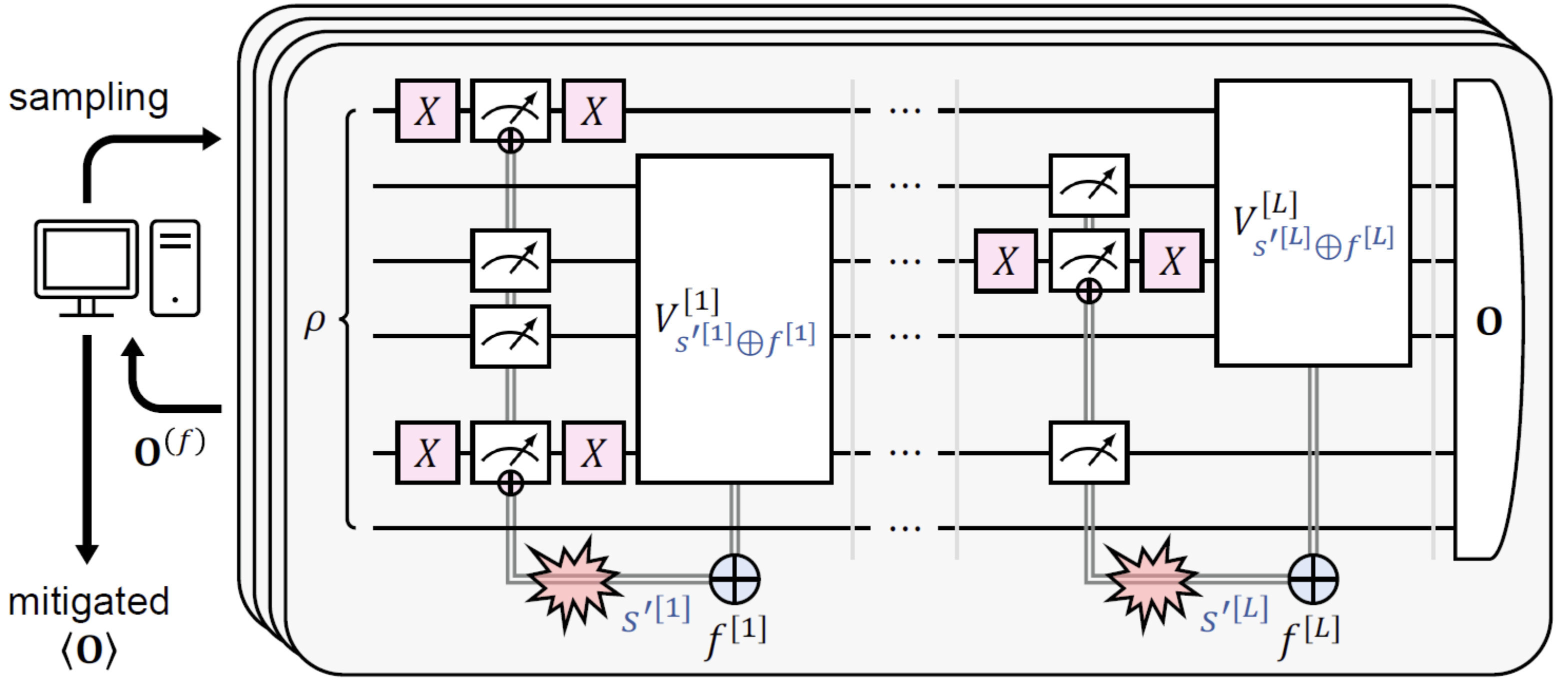

Standard methods for terminal readout error mitigation **not applicable**.

***Broken correlations*** *between post-measurement states and future conditional operations…*

# The protocol

**Randomized gate twirling**—bit-flip averaging (BFA)—to symmetrize readout error channels.

Inject probabilistic **bit-flips in feedforward data** to average over engineered ensemble of quantum trajectories.




1. Calibrate symmetrized readout error rates $\mathbf{q}$.
2. Two rounds of fast Walsh-Hadamard transform to compute estimator coefficients $\boldsymbol{\alpha}$.
3. Probabilistic sampling over feedforward bitmasks $f$ over $\boldsymbol{\alpha}/\|\boldsymbol{\alpha}\|_1$.

# Go to prom with me?

Probabilistic readout error mitigation (**PROM**).

Eliminates effect of readout errors on expectation values of arbitrary observables on **dynamic circuits**.

## Good things about PROM

- Works for **any number of layers** of mid-circuit measurements and feedforward.
- **Zero** circuit depth and 2-qubit gate count cost.
- Mild sensitivity to calibration errors and error channel approximations.
- **Integrates** with error mitigation methods for quantum gate noise (PEC, ZNE, CDR, *etc.*).

# Costs of error mitigation

**Specializations** of the general protocol if we assume structure in the noise channels:

| Assumptions | Classical Resources | | | Overhead $\xi^2$ |
|---|---|---|---|---|
| | Init. Time | Init. Space | Sampling Time | |
| - | $\mathcal{O}(m \cdot 2^m)$ | $\mathcal{O}(2^m)$ | $\mathcal{O}(1)$ | $\lVert\boldsymbol{\alpha}\rVert_1^2$ |
| Independent errors between layers | $\mathcal{O}(m \cdot 2^{\overline{m}})$ | $\mathcal{O}(L \cdot 2^{\overline{m}})$ | $\mathcal{O}(L)$ | $\prod_{l=1}^{L}(\xi^{[l]})^2$ |
| Fully independent errors | $\mathcal{O}(m)$ | $\mathcal{O}(m)$ | $\mathcal{O}(m)$ | $\prod_{\ell=1}^{m}(1-2r_\ell)^{-2}$ |
| Uniform error | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(m)$ | $(1-2r)^{-2m}$ |

$L$ layers of mid-circuit measurements (MCMs) + feedforward

$\overline{m}$ max. MCMs per layer

$m$ total MCMs

readout error rate $r_\ell$ for $\ell^{\text{th}}$ MCM

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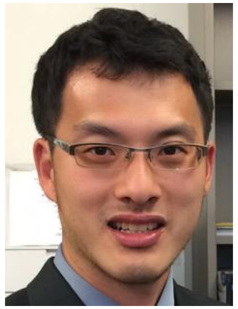

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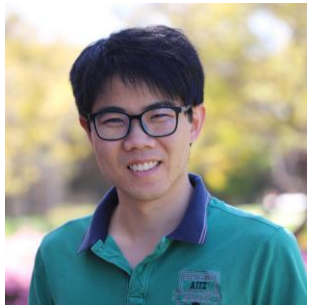

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? How can structure and physical constraints/symmetries be exploited to benefit QEC-QEM integration in quantum simulation use cases?

? What are the differences between a logical *vs.* a physical qubit that we can take advantage of, or need to deal with, when applying QEM on QEC-ed platforms?

# Happy to answer questions!

Error mitigation in quantum dynamics and condensed-matter simulations