algorithmiq

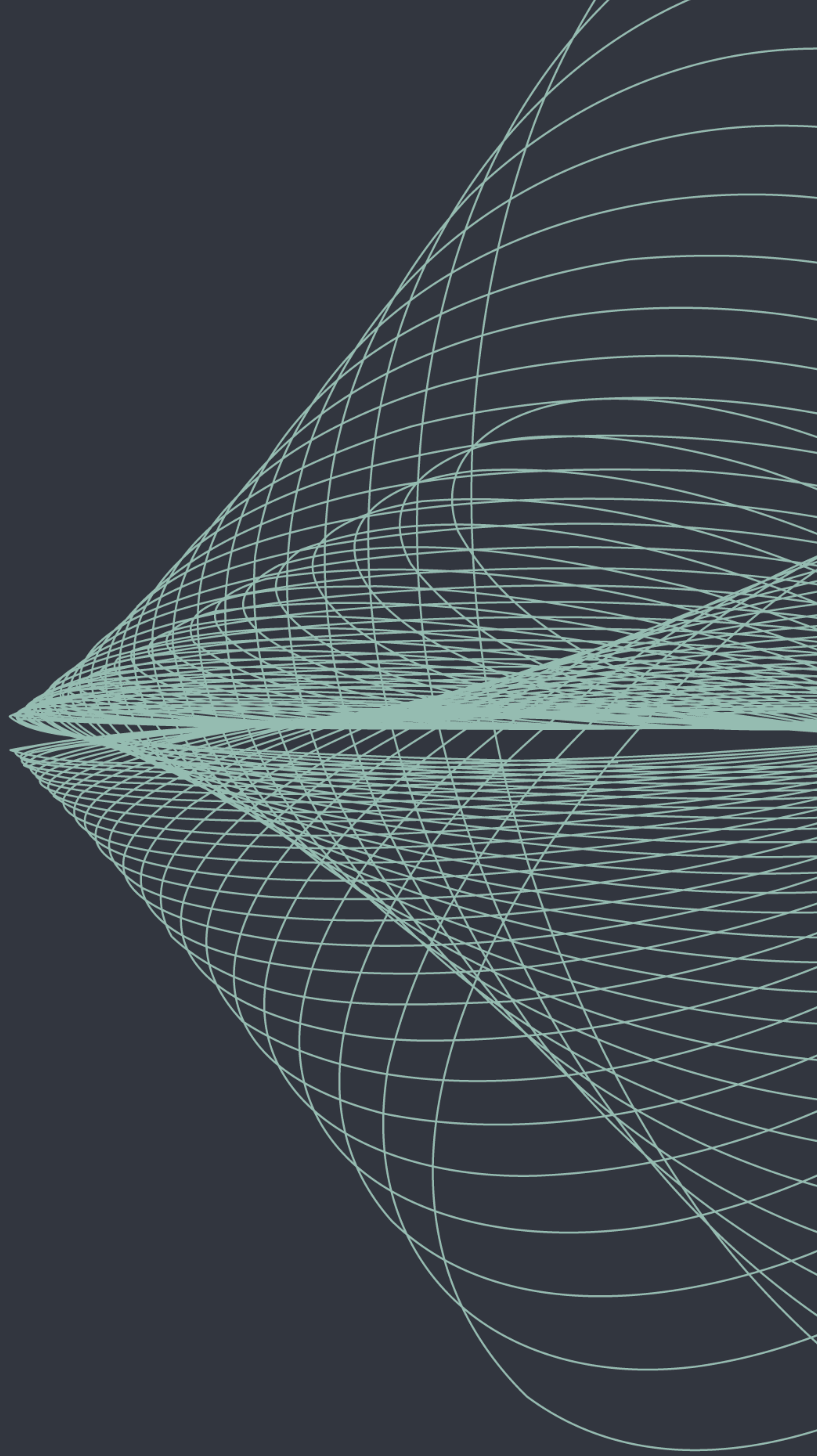
July 2025

TEM

A scalable tensor network based error mitigation

Matea Leahy

Junior Researcher



algorithmiq

Bringing quantum to life

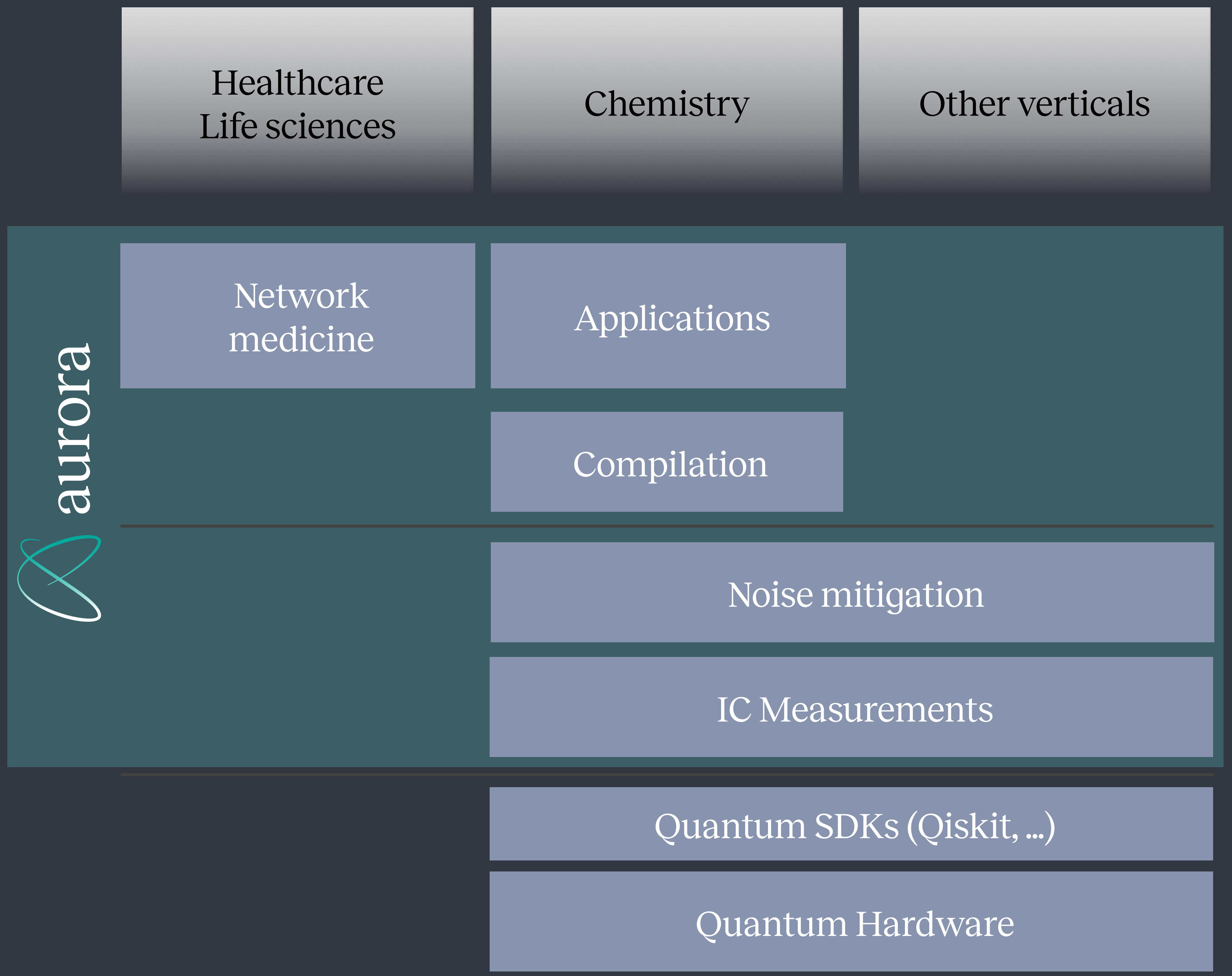
Our mission

We develop quantum software that makes quantum computers useful.

We use the unparalleled power of quantum computers for the fast and efficient cure and prevention of diseases.

Aurora

Full-stack software platform for drug design and discovery

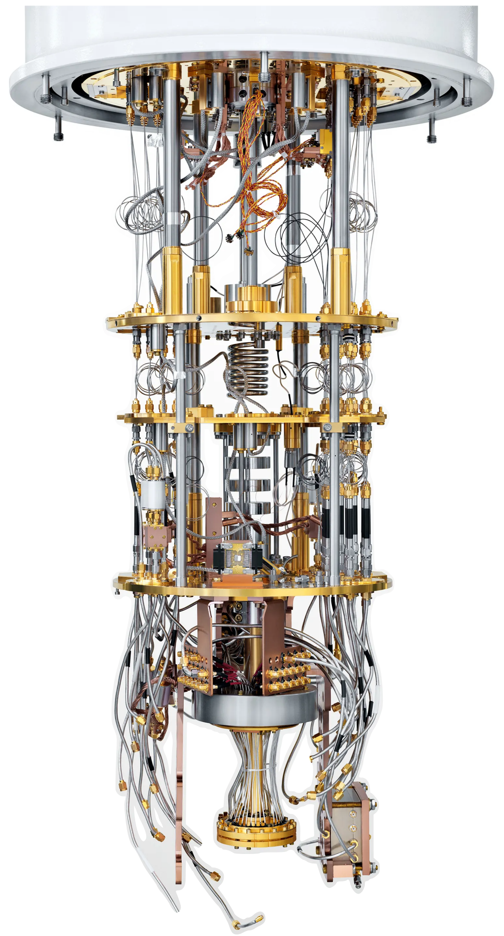


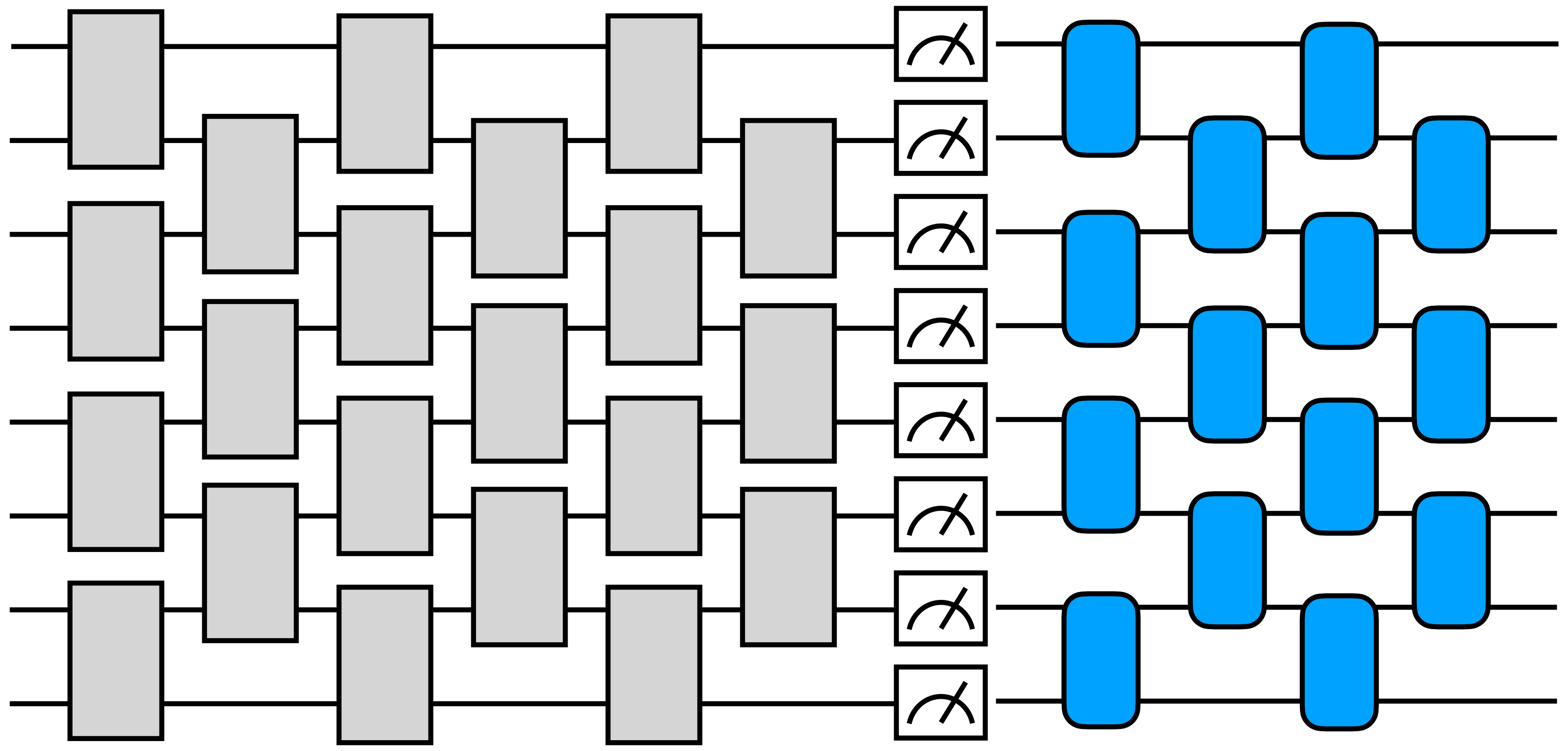
A hybrid approach

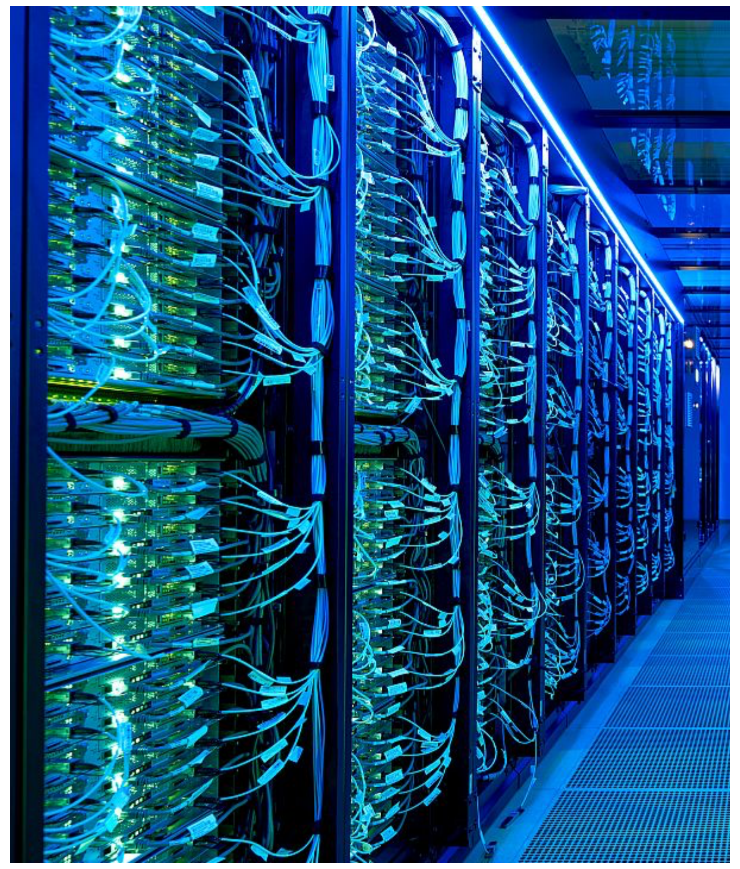
We prioritise combining quantum computing with tensor networks and high-performance computing (HPC)

Informationally complete measurements and tensor networks

Quantum computers + Tensor networks on HPC







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We develop methods built around informationally complete positive operator value measurements

Informationally complete measurements and tensor networks

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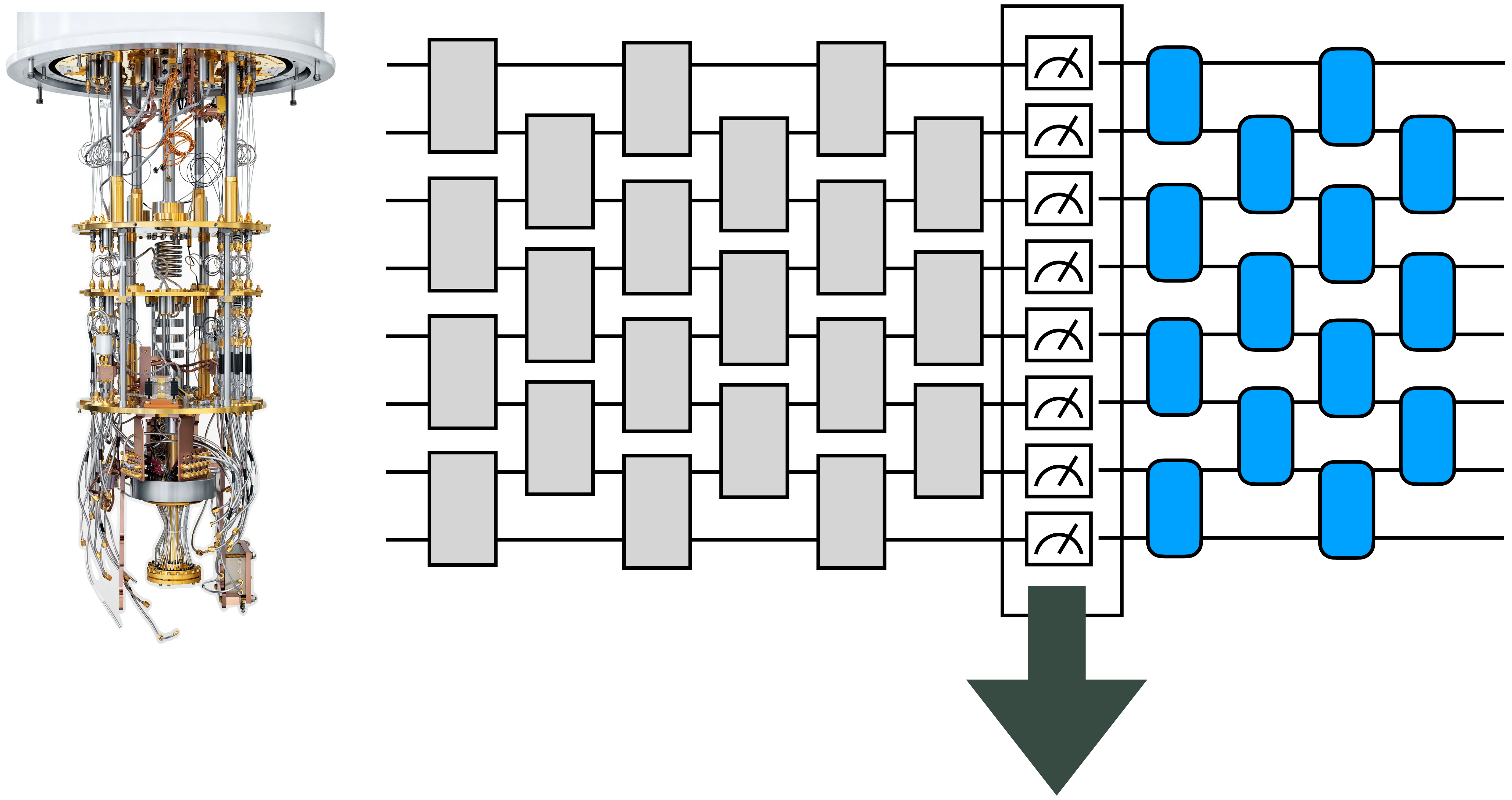

Informationally complete generalised measurements (IC-POVMs)

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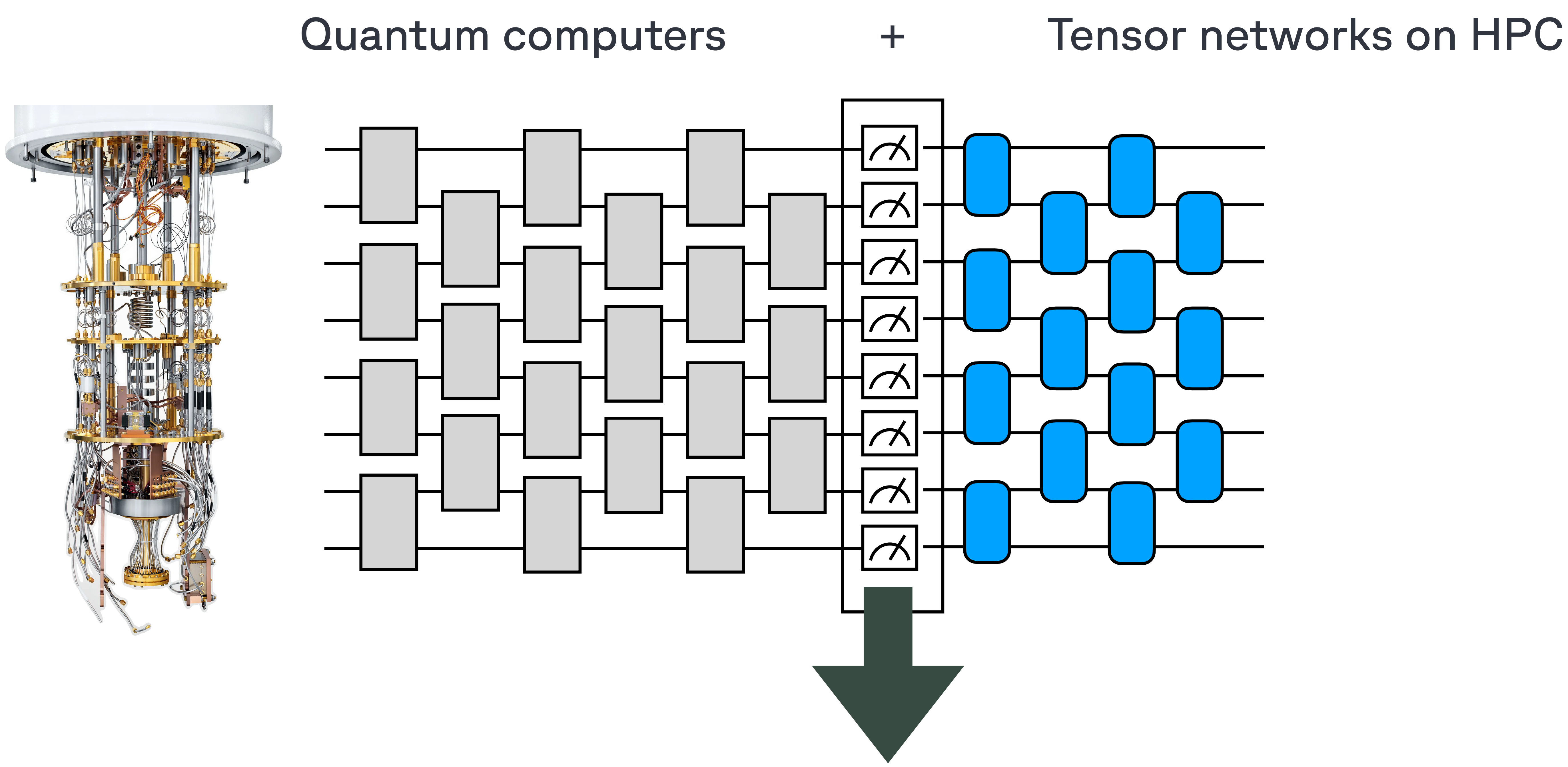

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- Provide shot efficient, unbiased estimators of the quantum state
- Can be optimised to extract more information
- Allow for linear transformations in post-processing

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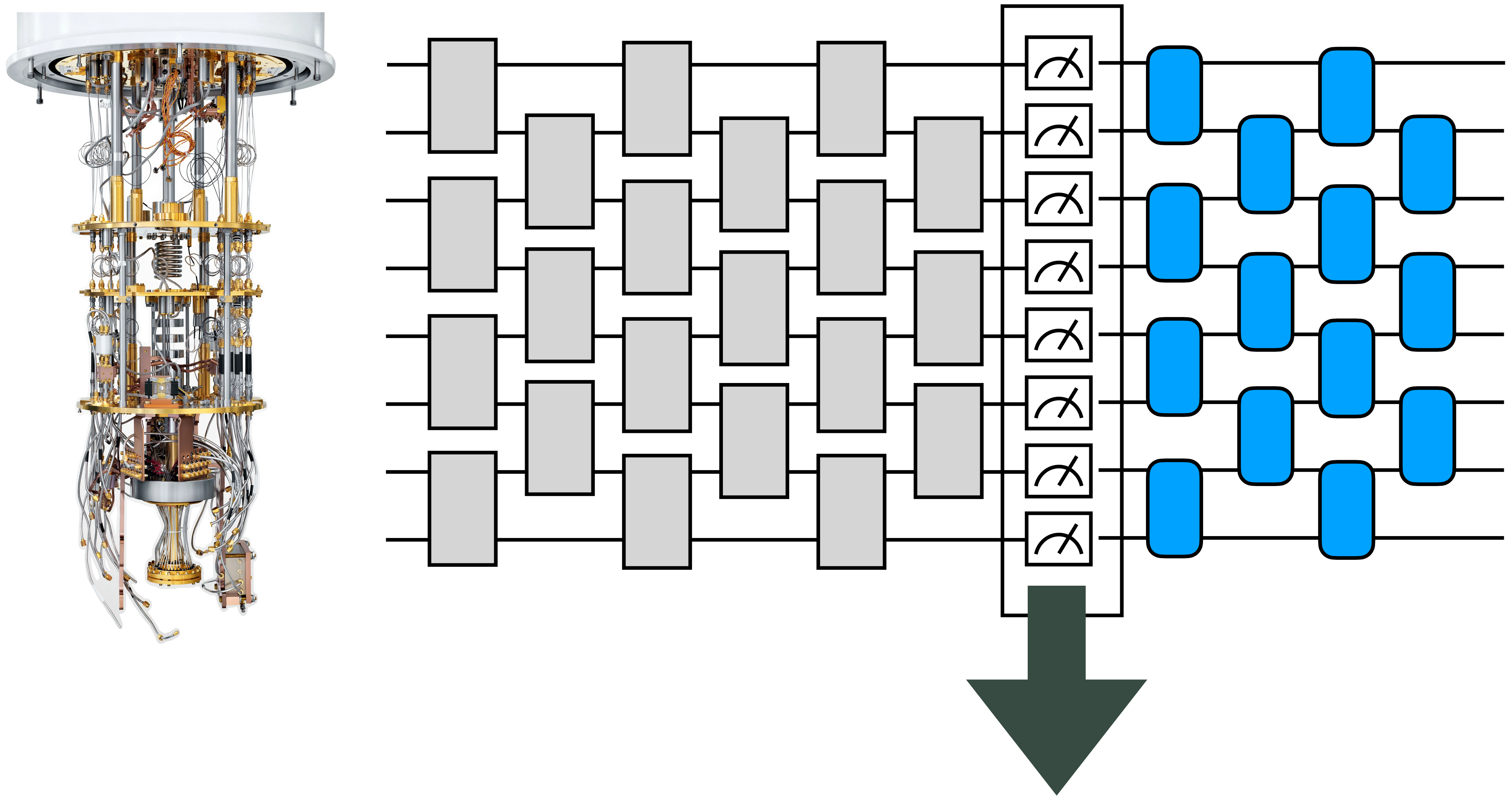

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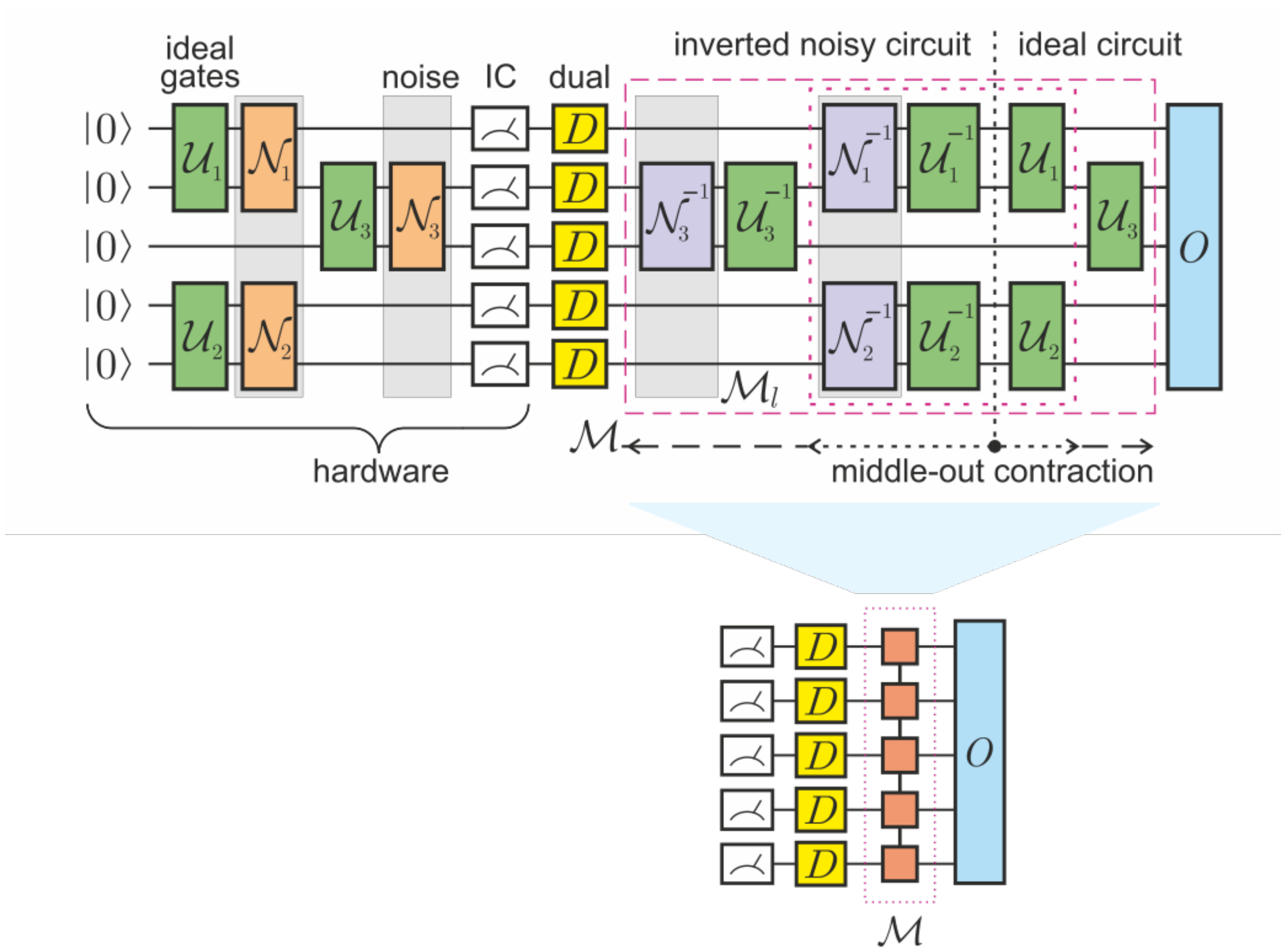
Scalable tensor network based error mitigation for near term quantum computing, Filippov 2023

Algorithm review

TEM

We build a tensor network that encodes the noise inverse map.

A scalable tensor network based error mitigation for near term quantum computing



- Noise mitigation map in software post-processing
- Tensor network noise mitigation method, computationally easier as the noise decreases
- A tensor network encodes the inverse of the noise map (cheaper than simulating the whole circuit)

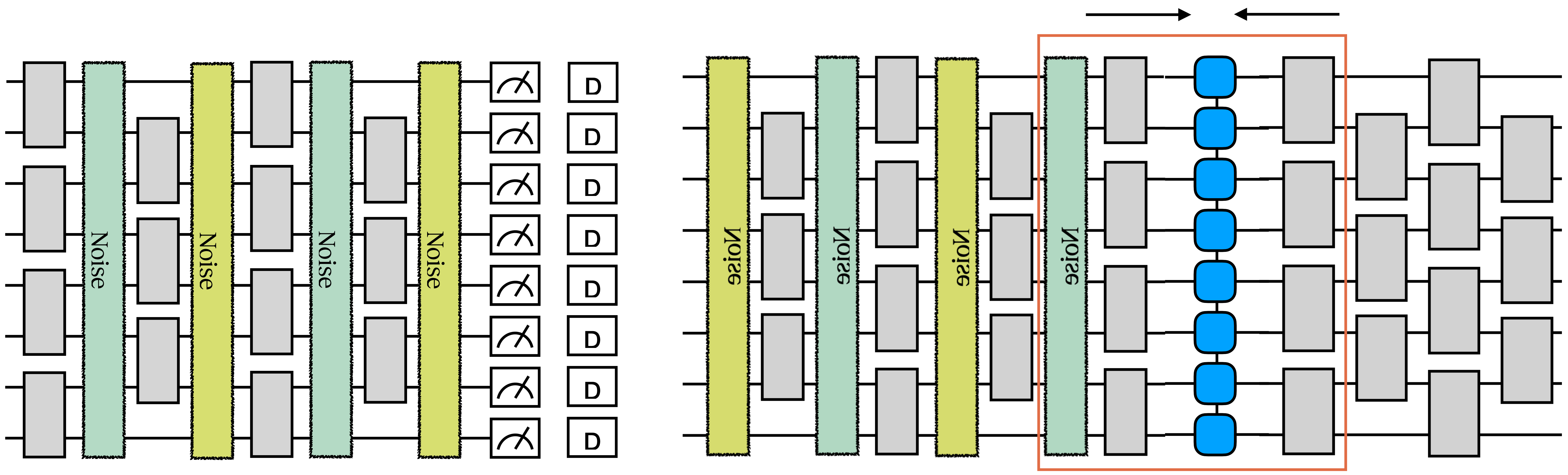
+

Noise Assumptions:

- Not necessarily local
- Small (consistent with existing hardware and constantly improving)
- Known/Efficiently representable

Middle-out contraction

We contract from the middle outward, building our noise inverse map as a matrix product operator



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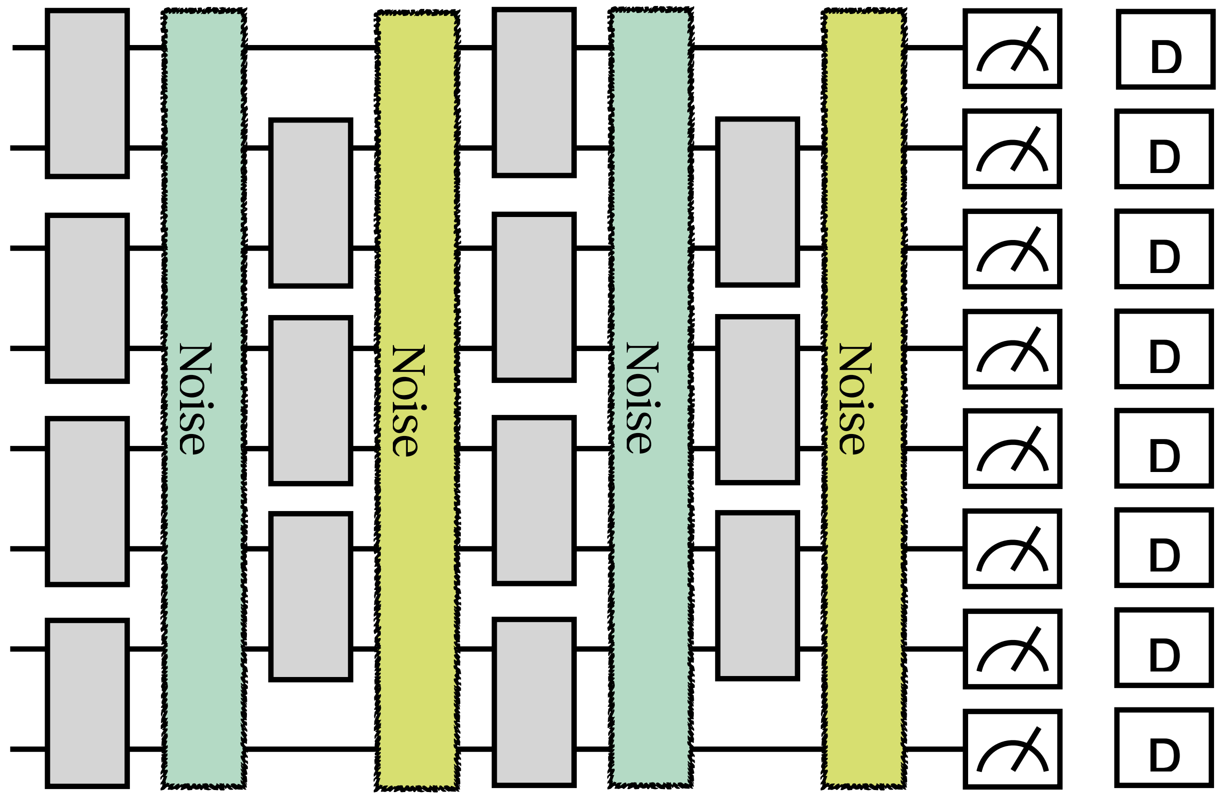

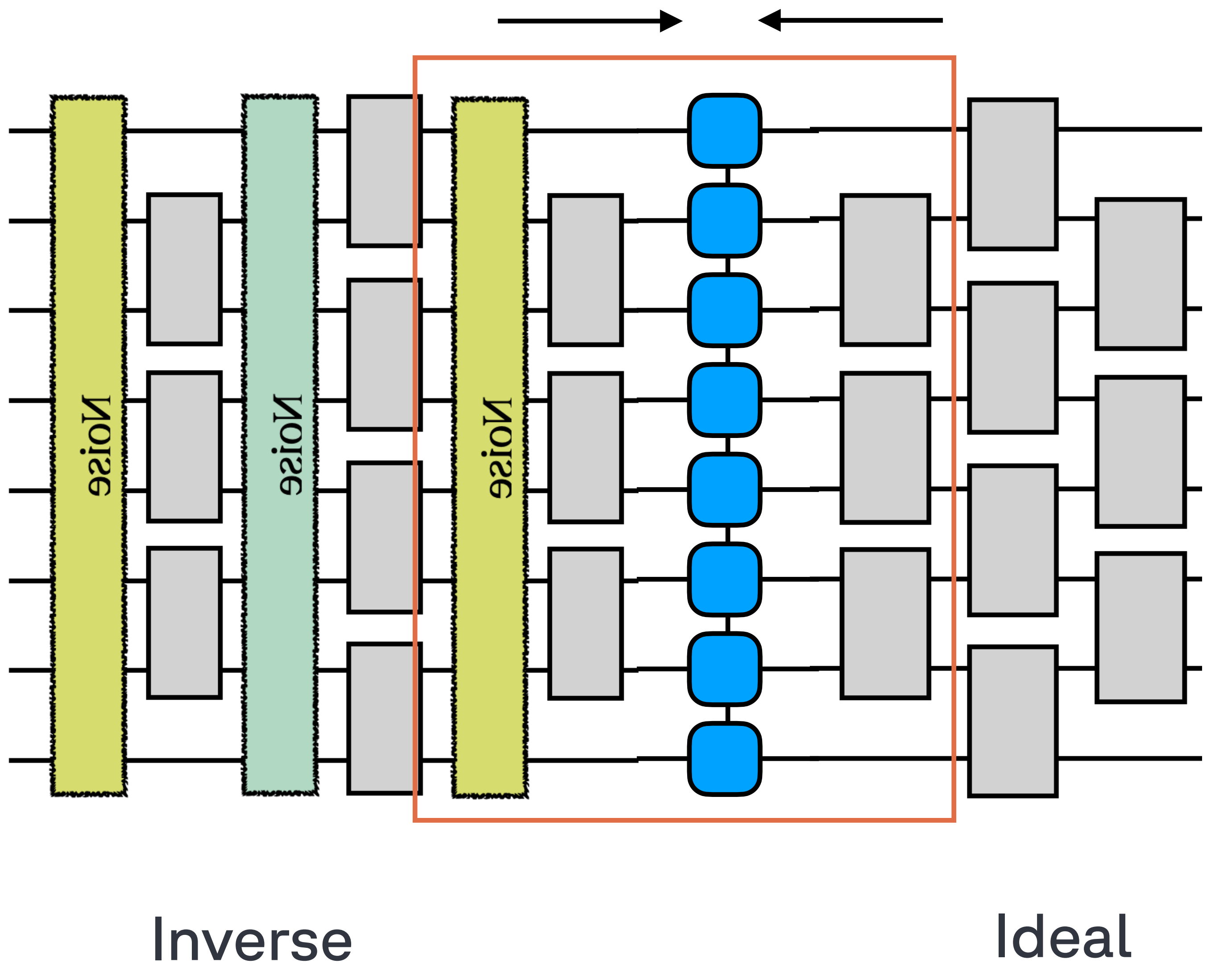

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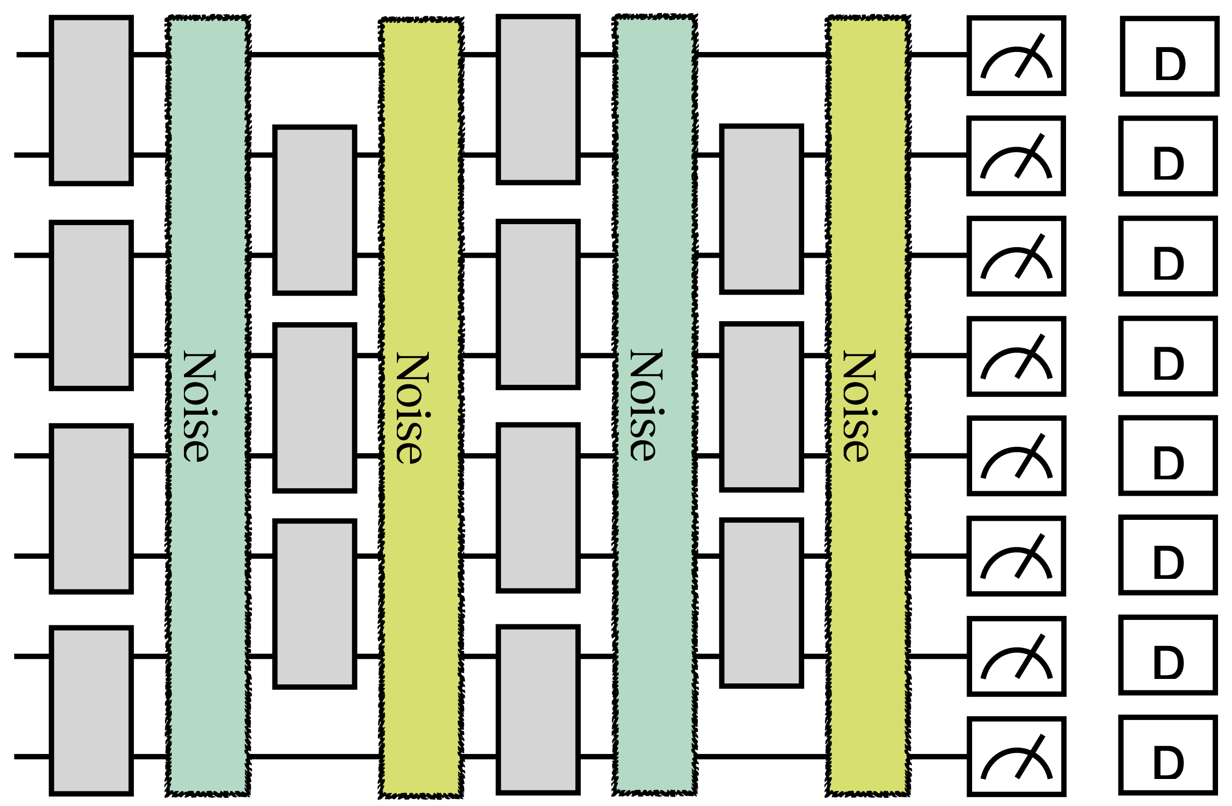

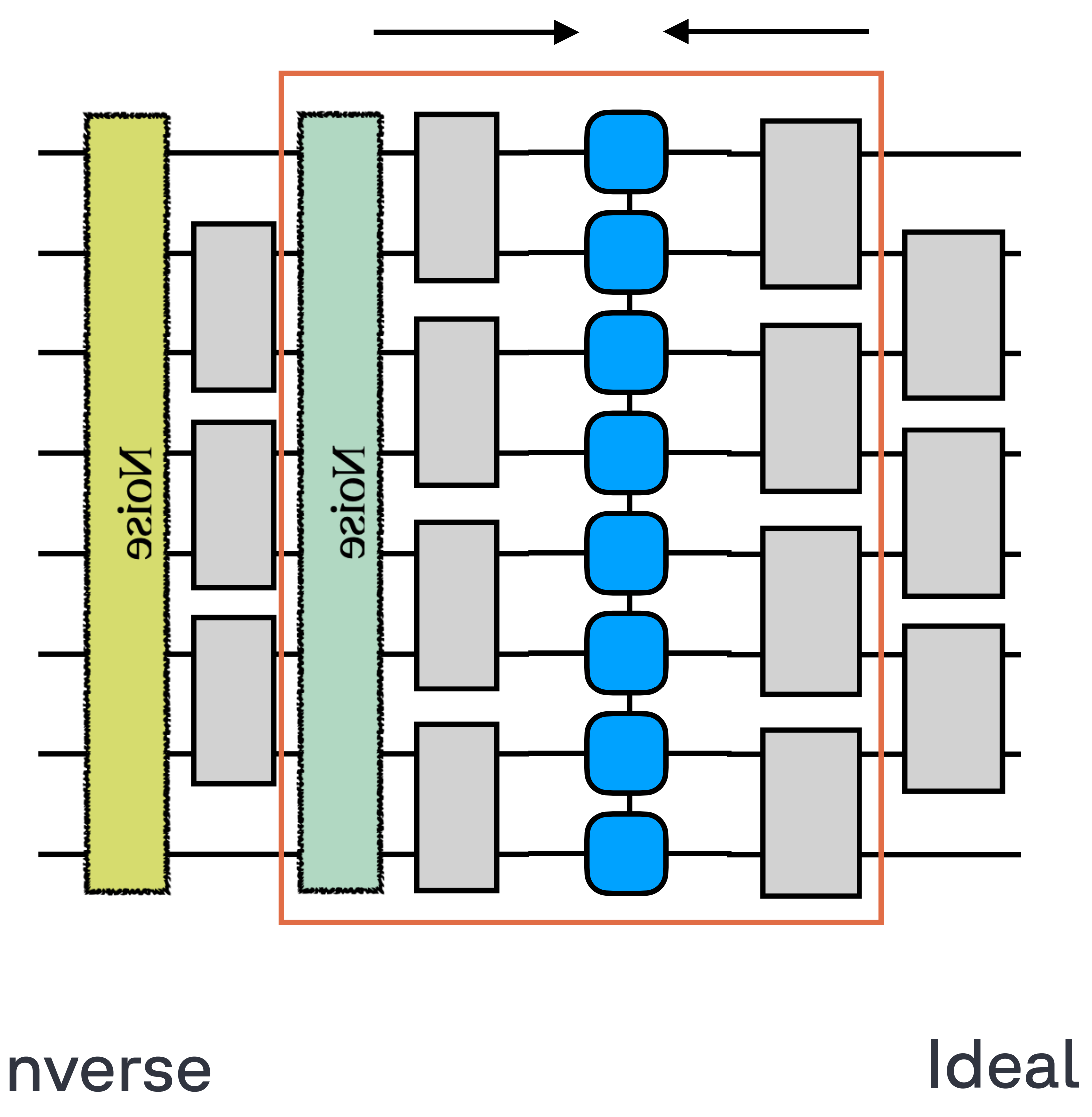

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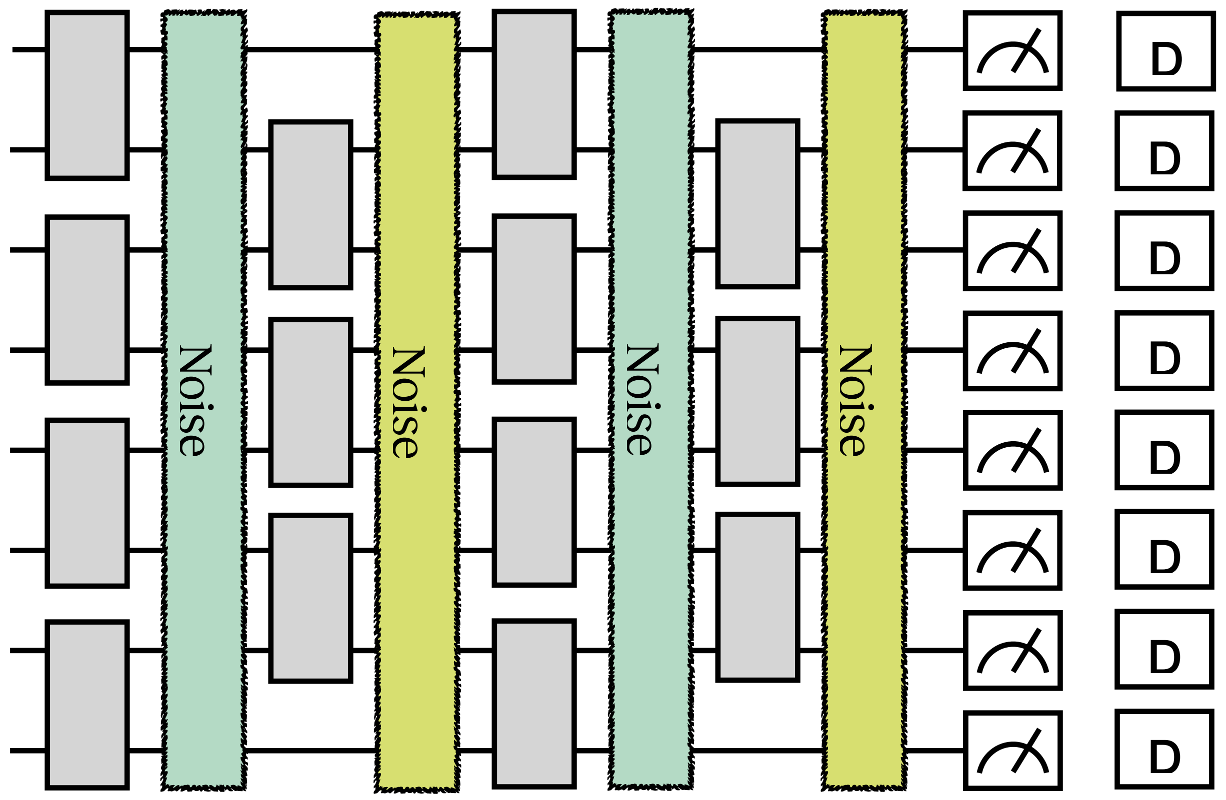

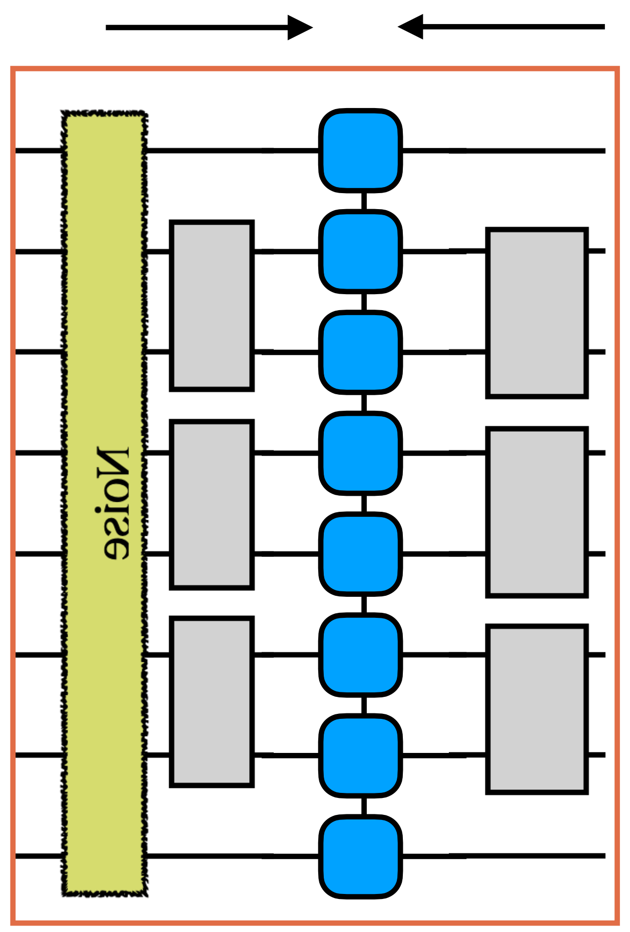

Inverse Ideal

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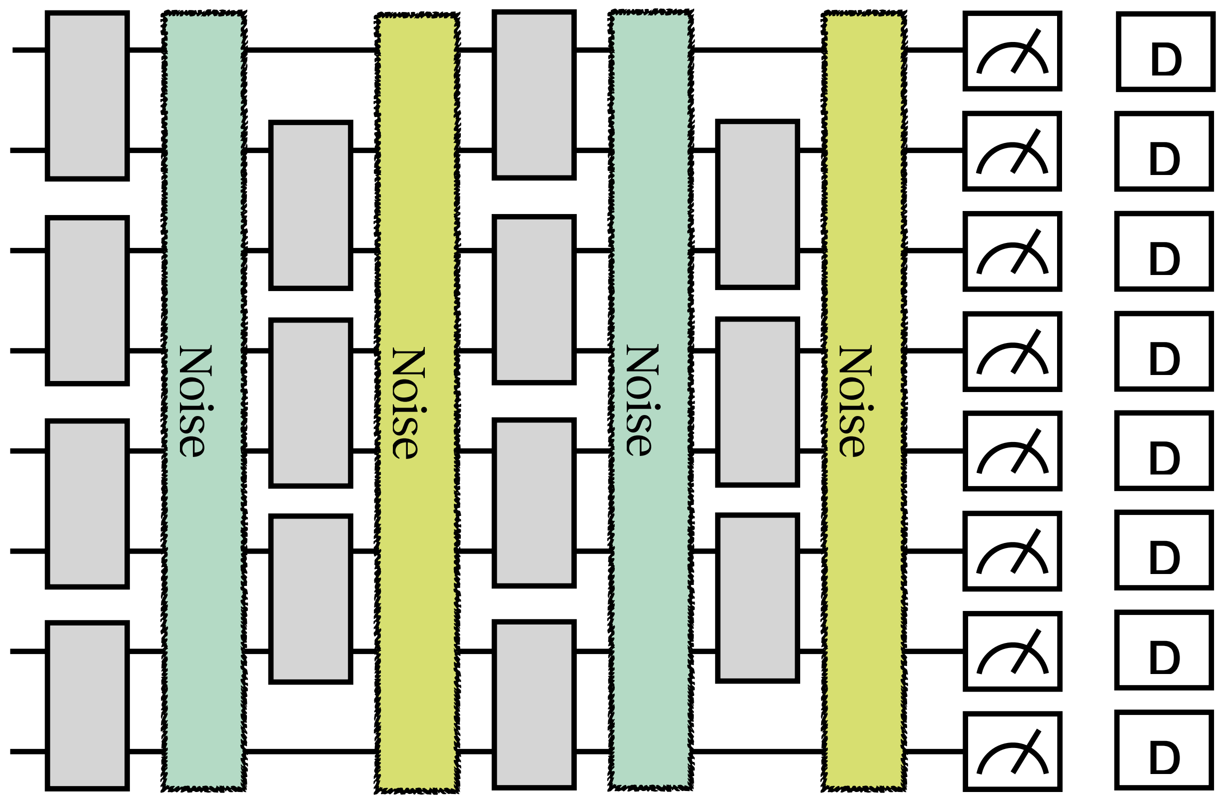

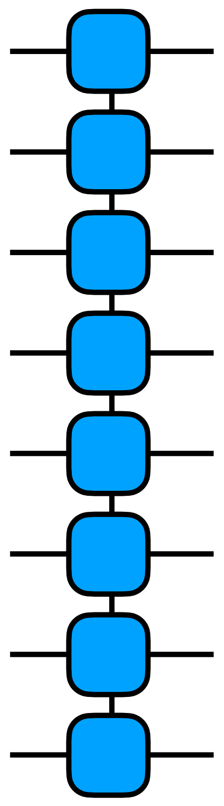

$\mathcal{N}^{-1}$

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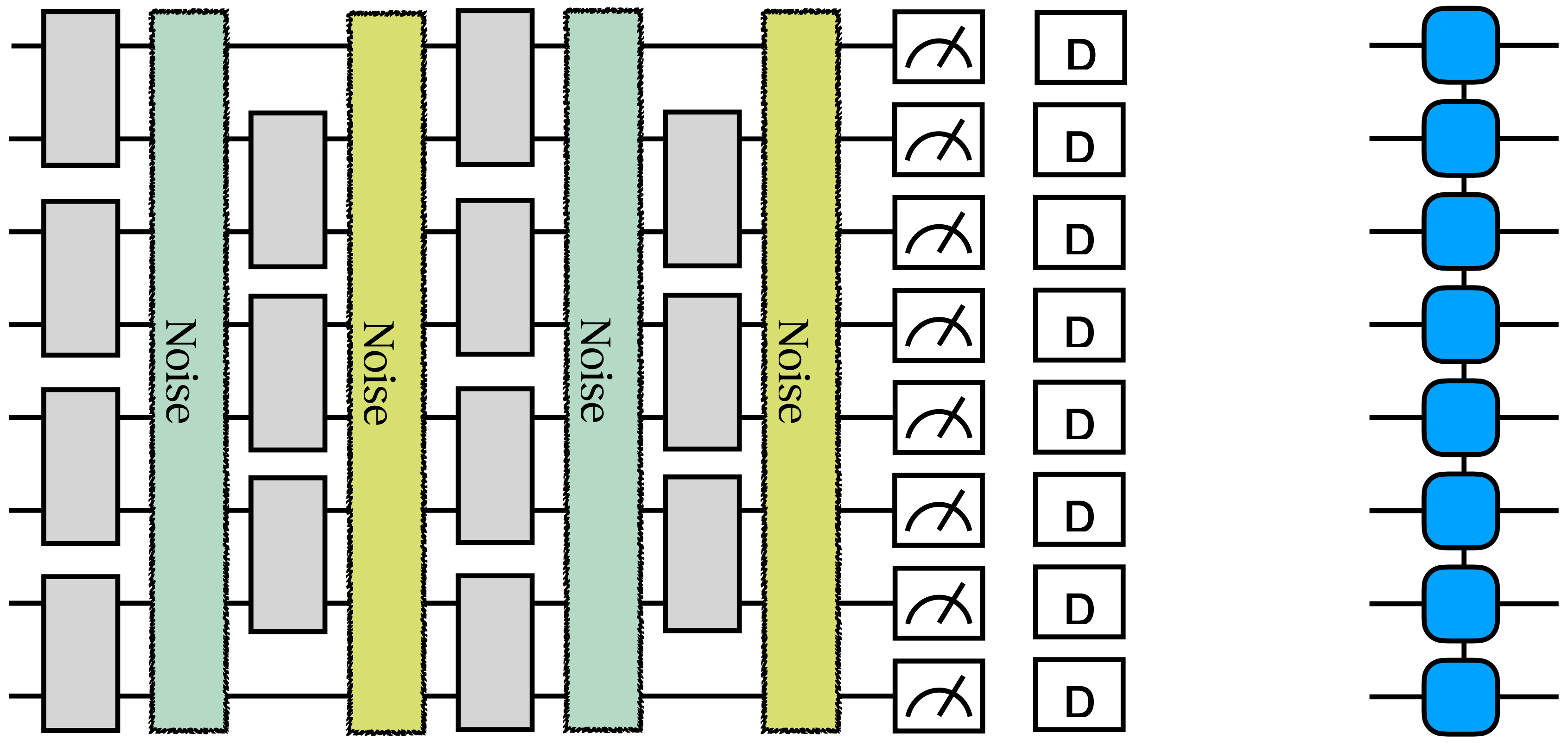
Truncation

Untreated, the bond dimension of the MPOs would grow exponentially in the number of layers.



$\mathcal{N}^{-1}$

The MPO is compressed after each iteration either to a fixed bond dimension or to a desired precision.

This is achievable using the smallest singular values in the canonical representation of the MPO or by variational means

$$\mathcal{N} \approx Id + \epsilon\Lambda$$

- MPO compression error is at most linear in ϵ
- MPO compression cost is cubic in bond dimension

Noise characterisation

Capture gate noise, crosstalk and decoherence using noise characterisation

Represent the noise channel with a sparse Pauli Lindbladian (SPL) noise model

$$\mathcal{N} = e^{\mathcal{L}} \quad , \ \mathcal{L} = \sum_i \lambda_i (P_i \rho P_i^\dagger - \rho)$$

van den Berg, E., Minev, Z.K., Kandala, A. 2023

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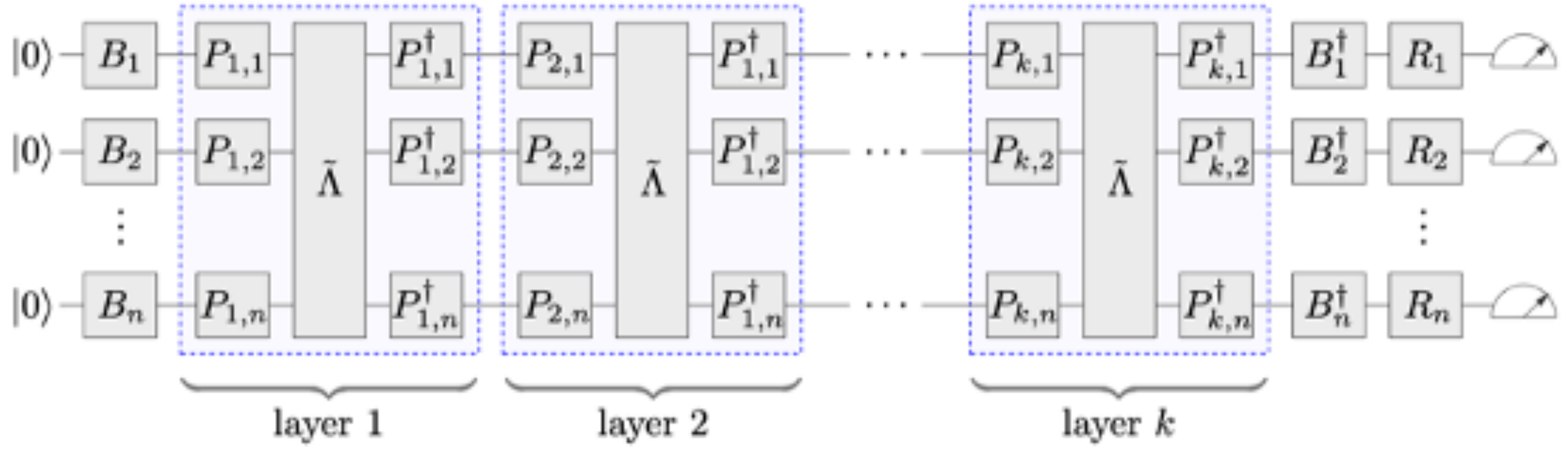
Pauli twirling employed to bring into Pauli form



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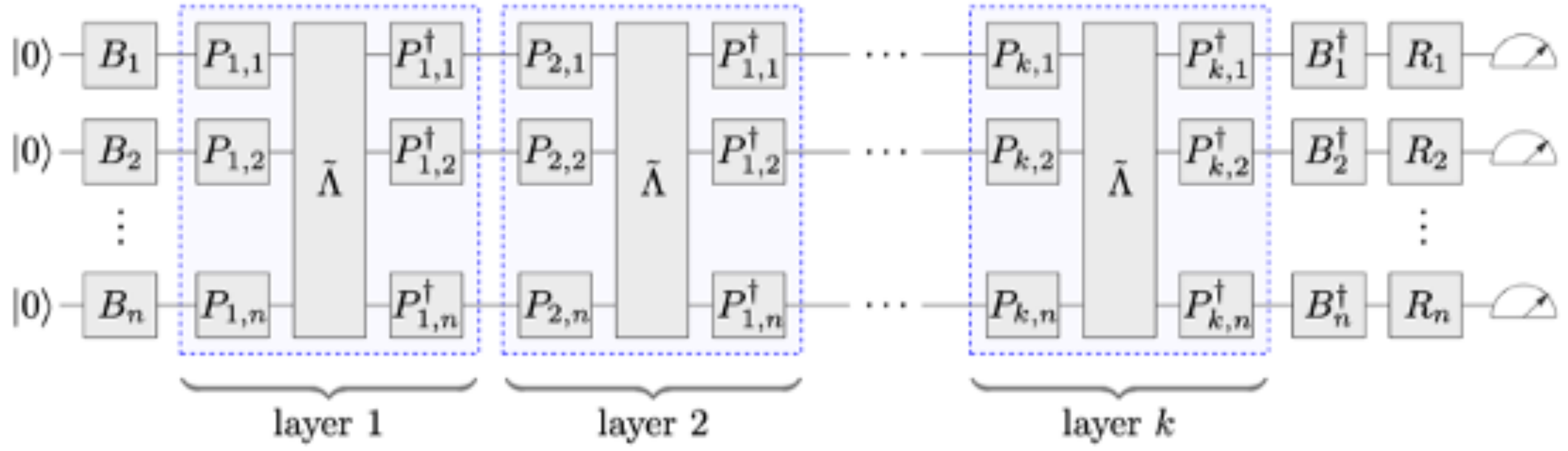
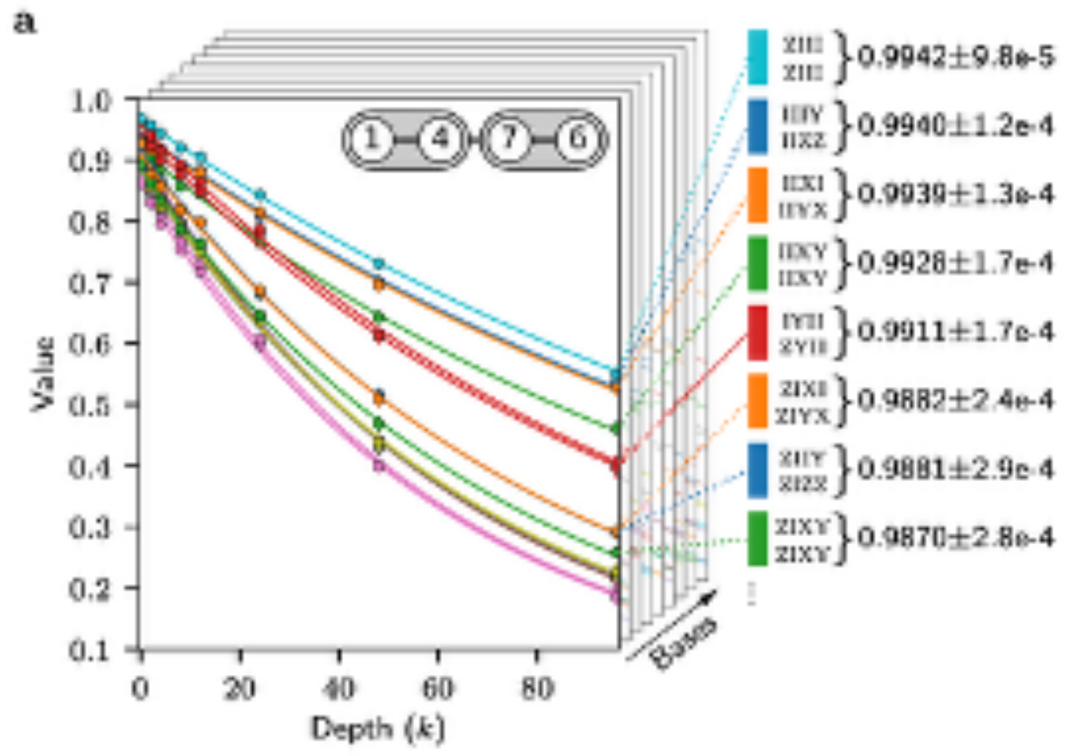


λ_i are learned through cycle benchmarking



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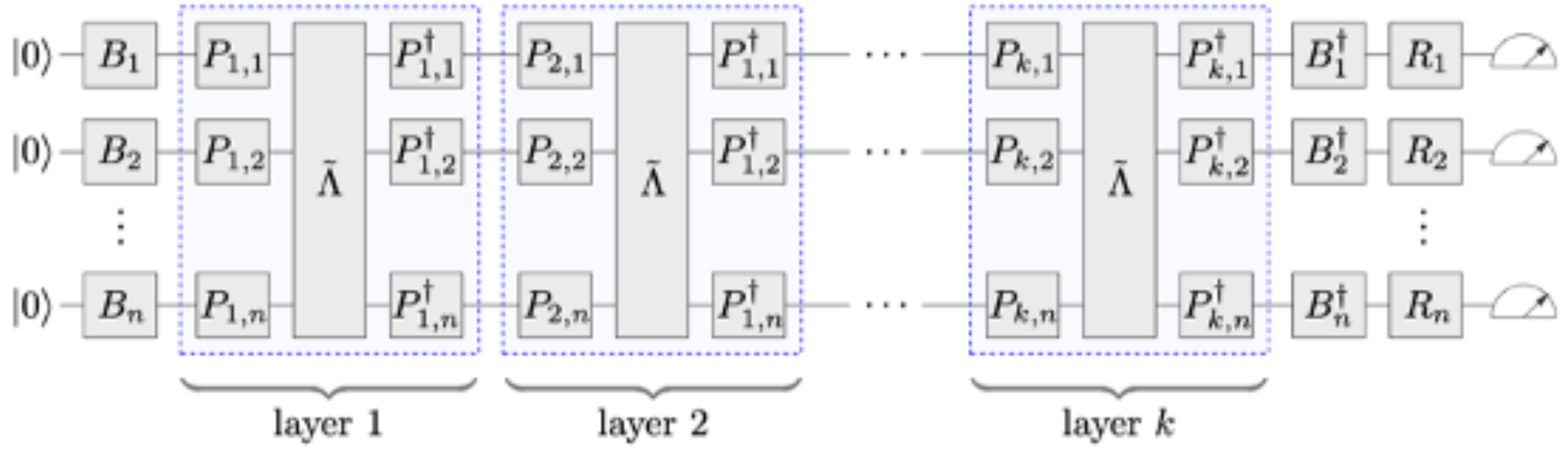

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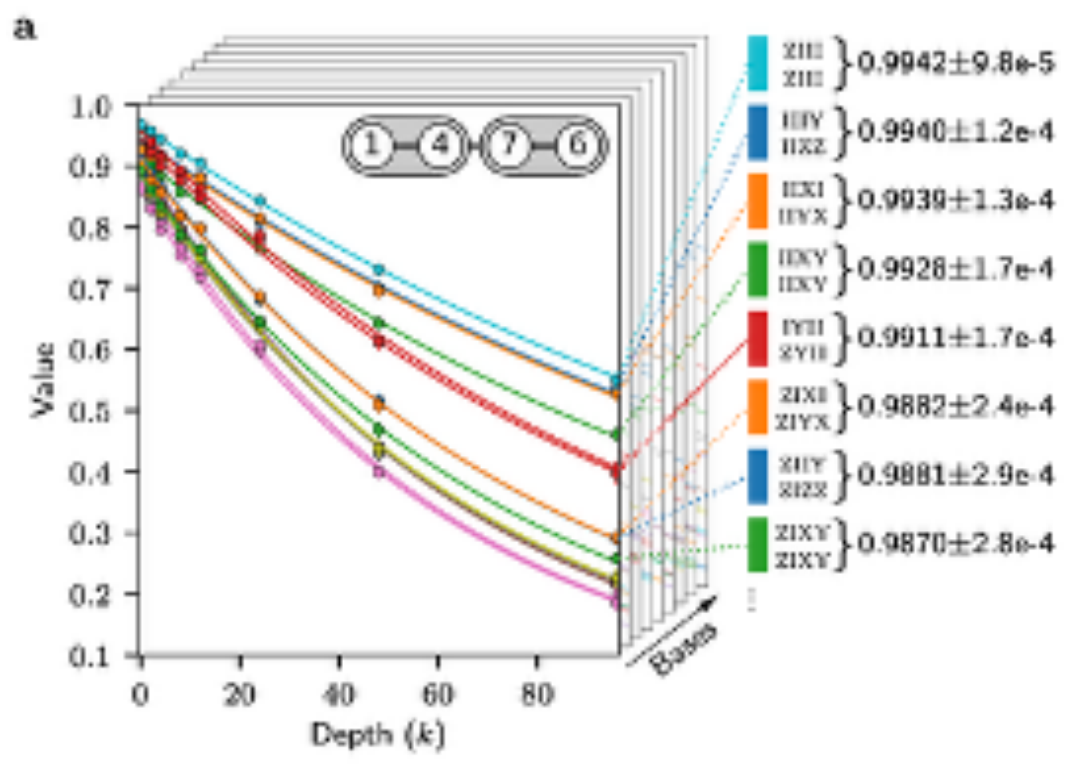
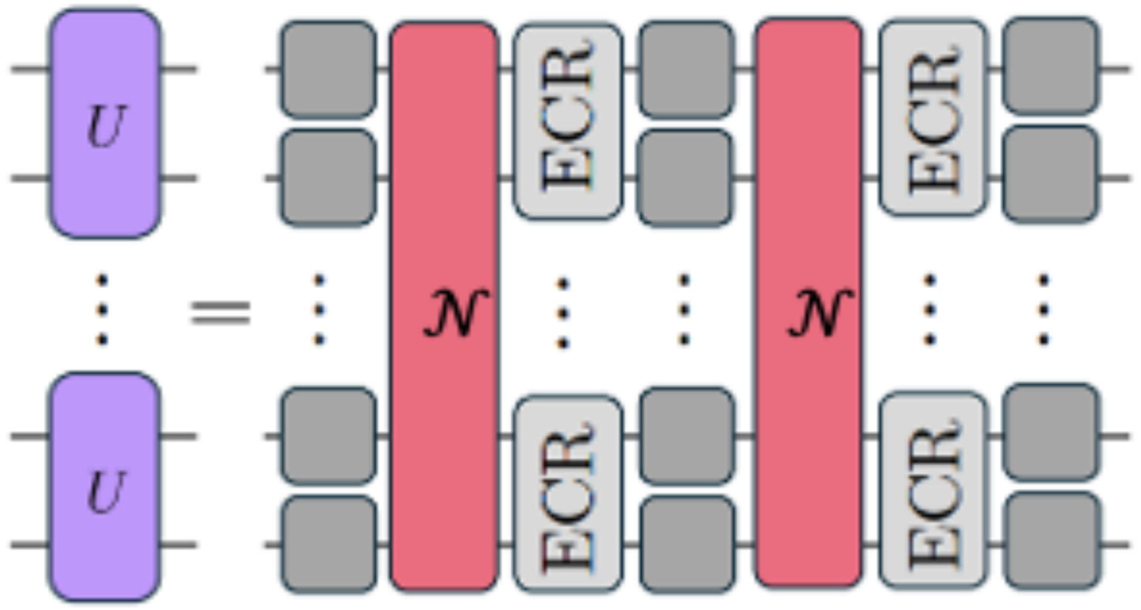


Each layer in the circuit is accompanied by it's own learned noise channel



To name a few

Crucial for current state of the art noise mitigation

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Probabilistic Error Cancellation

$$O^{ideal} = \sum_i \eta_i O_i^{noisy}$$

η_i learned from a quasi-probability distribution

The ideal circuit is sampled from a quasi-distribution of noisy ones

Unbiased

van den Berg, E., Minev, Z.K., Kandala, A. 2023

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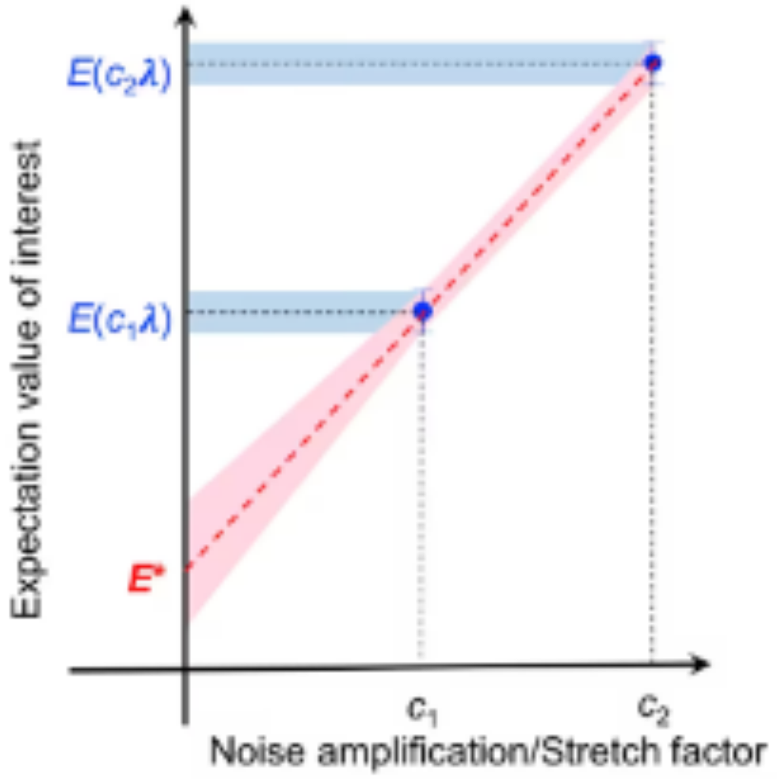
van den Berg, E., Minev, Z.K., Kandala, A. 2023

Zero Noise Extrapolation



Intentionally amplify the noise then fit and extrapolate.

Biased, particularly for deep circuits

Kim, Y., Eddins, A., Anand S., 2024

Measurement overhead

How many additional shots do we need to achieve the same precision when performing error mitigation?

Sampling overhead:

$$\Gamma = \frac{N_{more\ shots}}{N_{shots}} = \frac{(\Delta O)^2_{mitigated}}{(\Delta O)^2_{noisy}}$$

Adapted from:
Filippov, Maniscalco, García-Pérez, arXiv:2403.13542

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$$\Gamma_{ZNE} \approx (1 + 1.795\epsilon NL)^2 e^{\epsilon NL} \ , \ \ \Gamma_{PEC} \approx (1 + 2\epsilon)^{NL} \approx e^{2\epsilon NL} \ , \ \ \Gamma_{TEM} \approx (1 + \epsilon)^{NL} \approx e^{\epsilon NL}$$

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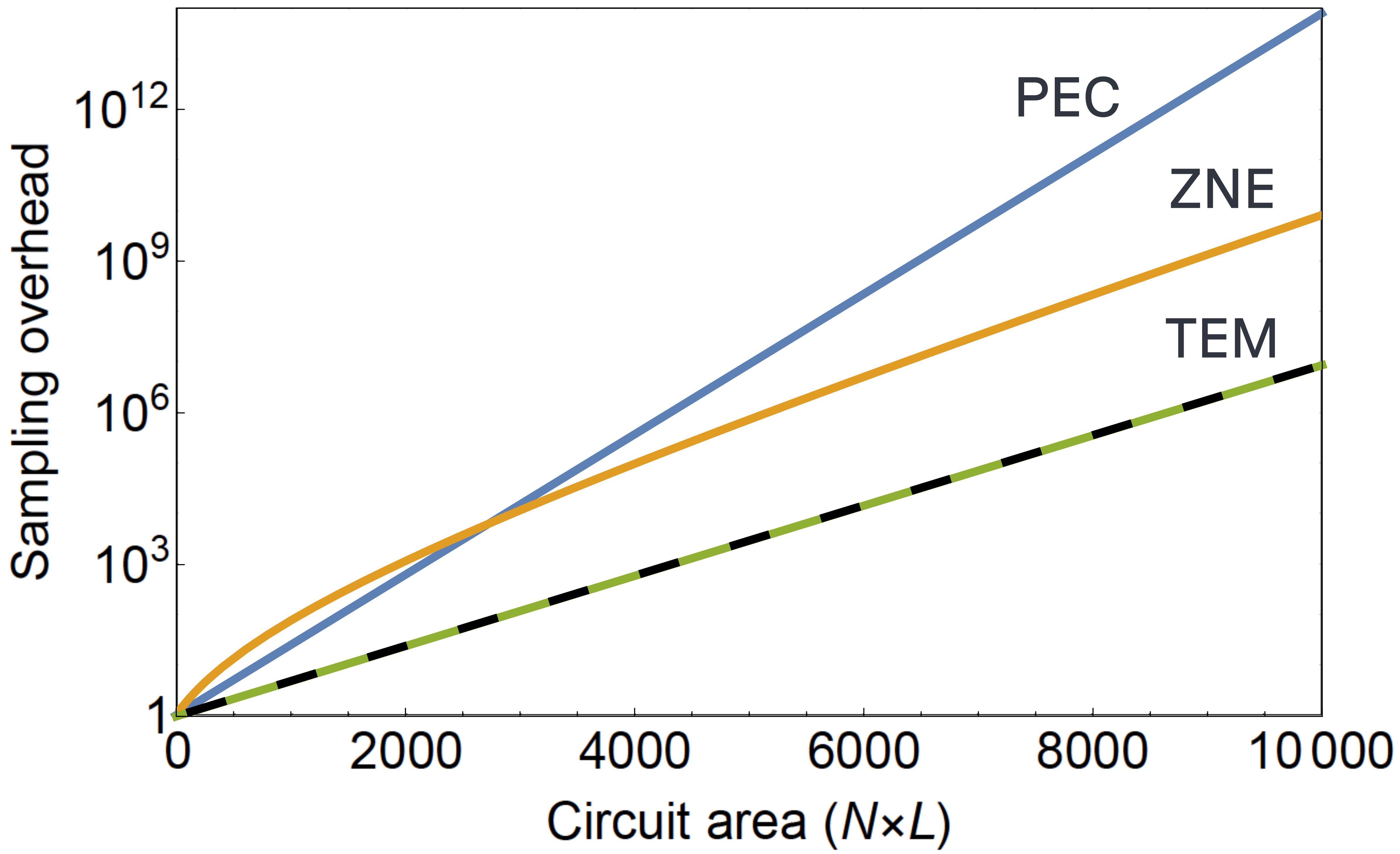
Measurement overhead

TEM saturates the theoretical lower bound for unbiased error mitigation



Assumptions:

- High weight Pauli observables
- Dense NxL quantum circuits
- Error/qubit/gate/layer = 0.16%

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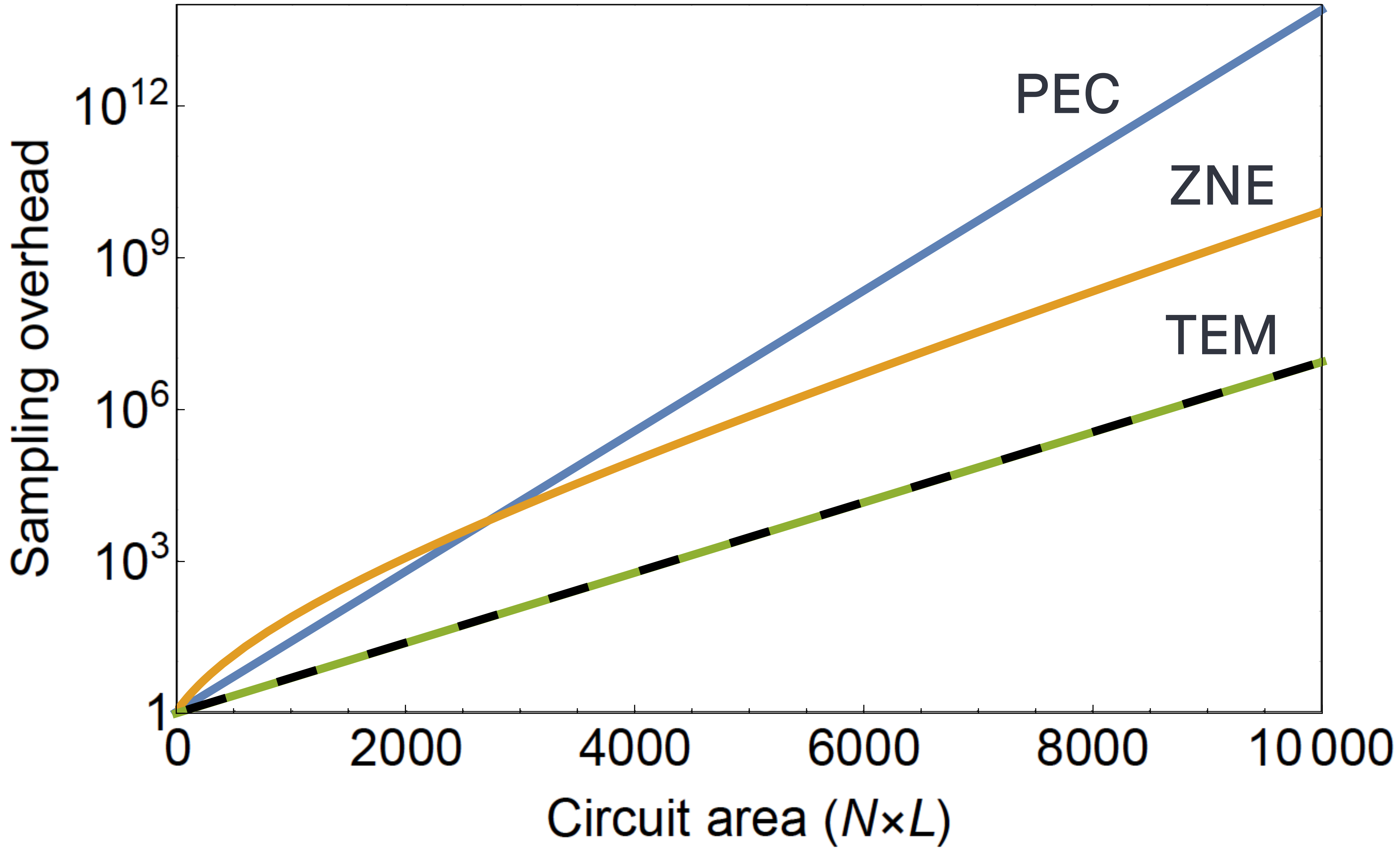

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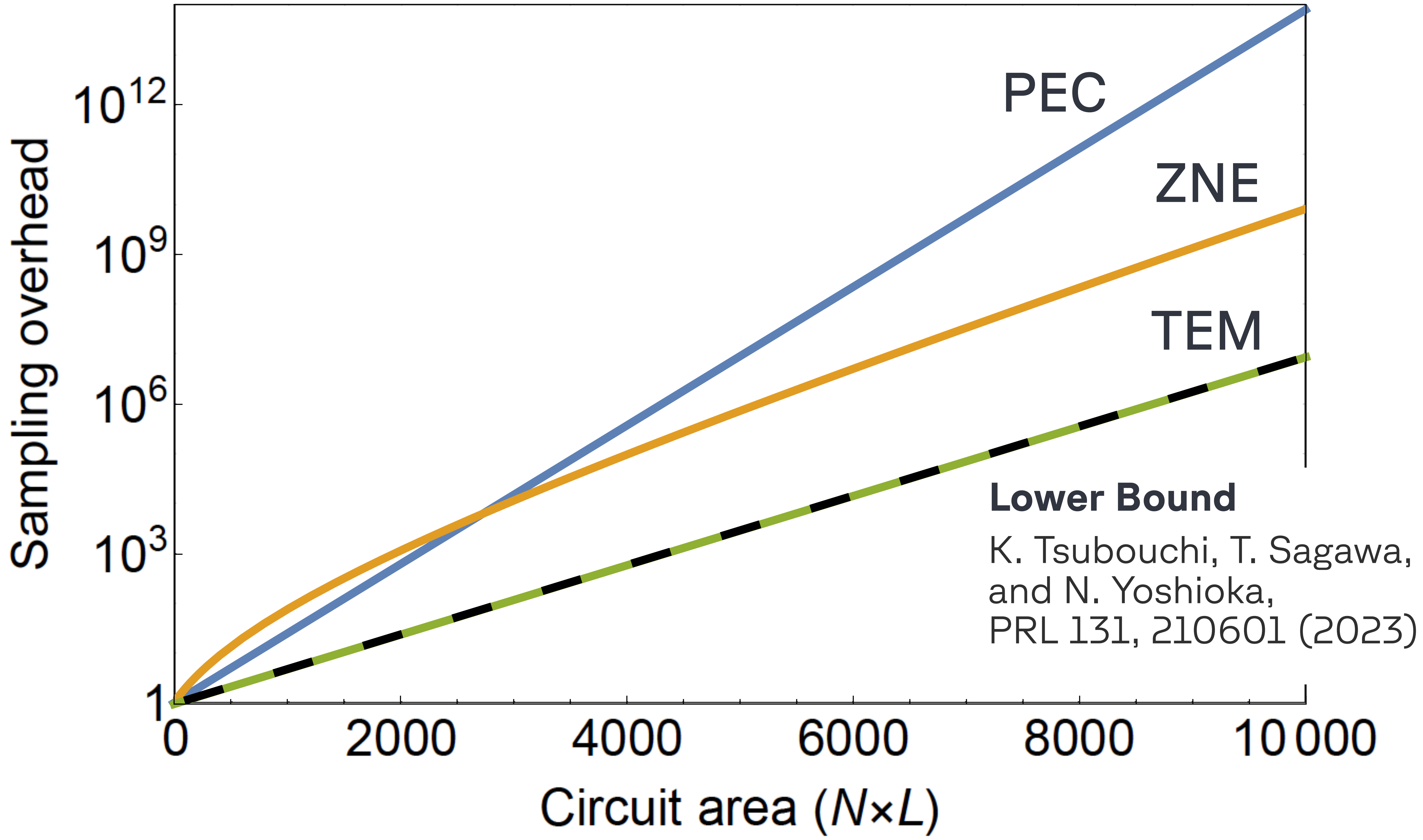

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- Theoretical lower cost bound for sampling overhead shown as the dashed black line.

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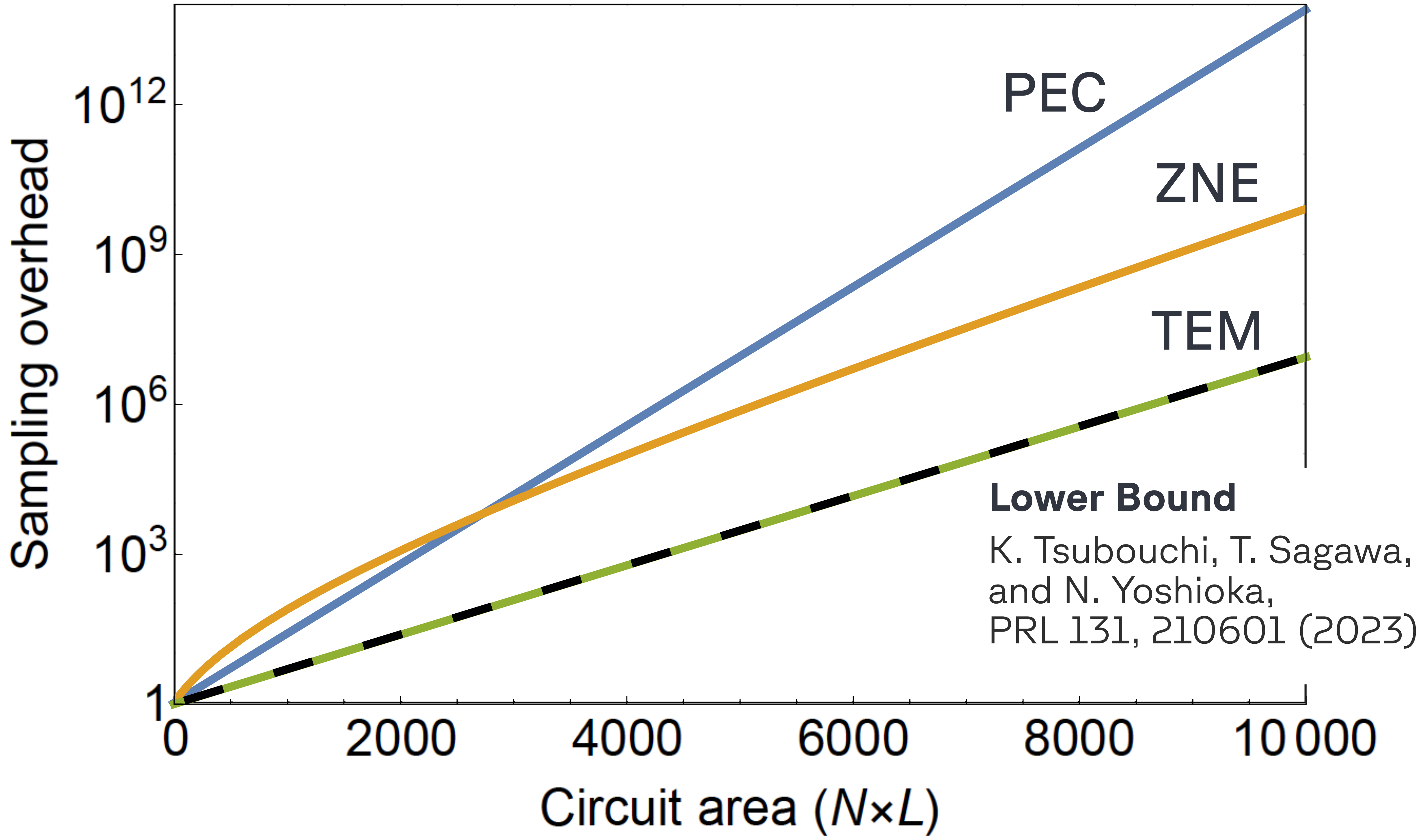

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- TEM saturates the lower bound!

Adapted from:
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Utility scale demonstration

algorithmiq

IBM

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Dynamical simulations of many-body quantum chaos on a quantum computer*

(91 qubits, 91 brickwork layers, 4092 CNOTs)

*In collaboration with Ivano Tavernelli's group at IBM Zurich, John Gold's group at Trinity College Dublin and Abhinav Kandala's team at IBM Yorktown.

Why is this interesting?

1

Interesting physics

—

Quantum dynamics of the kicked Ising model in a transverse field.
A playground to study many body physics.

2

Dual Unitary circuits

—

Quantum circuits comprised of two qubit gates that are unitary in both temporal and spatial directions

3

A benchmark for quantum simulation

—

Analytical solution exist for specific points in parameter space which can be used as a benchmark.

4

Noise model calibration

—

Solvable points can be used to further calibrate noise models

5

Ideal for showcasing error mitigation

—

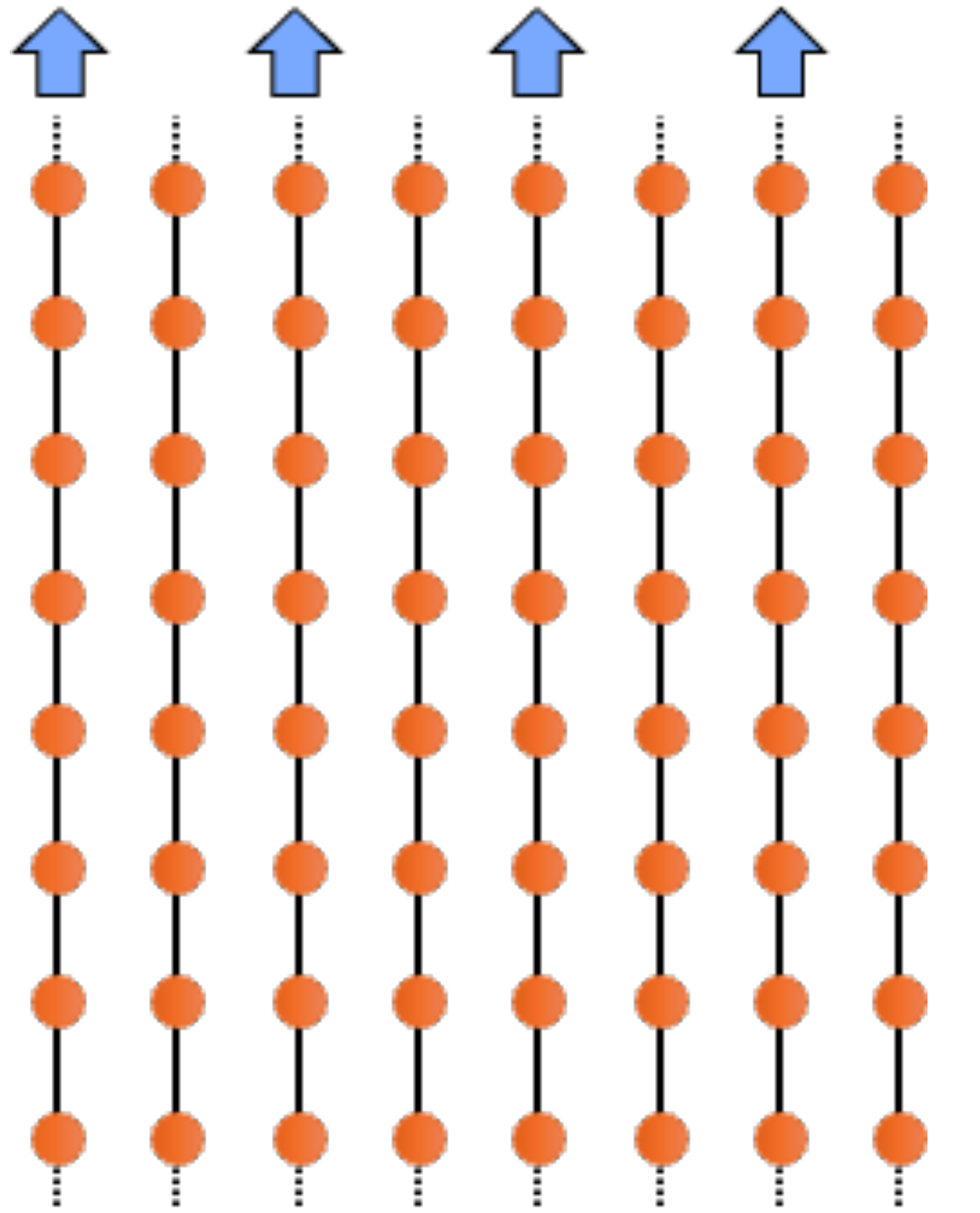
These pieces combine to provide an excellent test bed for noise mitigation methods!

Model

Ising spin chain with periodic transverse field kick

Ising: $$H_I = J \sum_{n=0}^{N-2} \sigma_n^z \sigma_{n+1}^z + h \sum_{n=0}^{N-1} \sigma_n^z$$

Kick: $$H_K = b \sum_{n=0}^{N-1} \sigma_n^x$$



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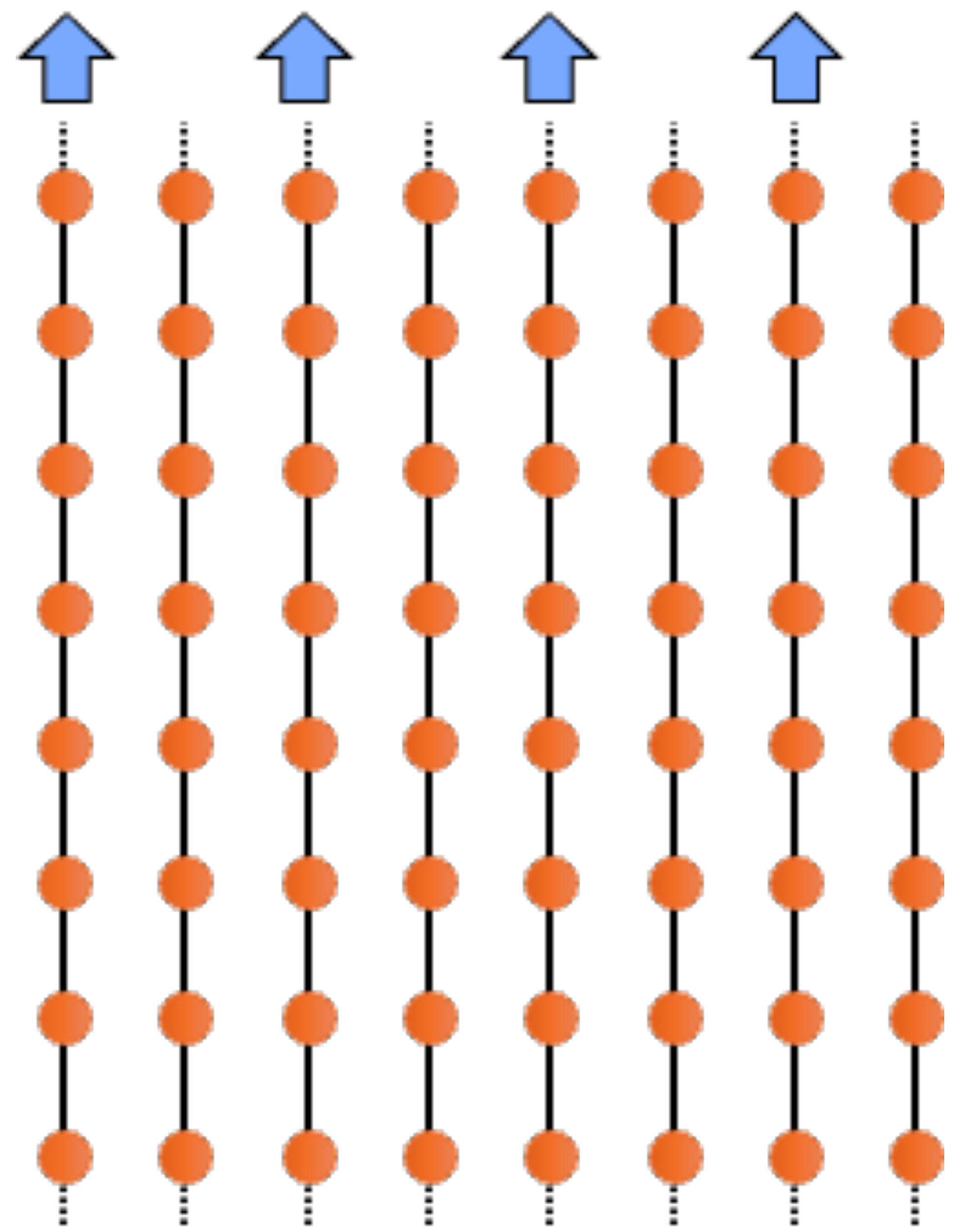
Hamiltonian: $$H_{KI}(t) = H_I + \sum_{m \in Z} \delta(t-m) H_K$$



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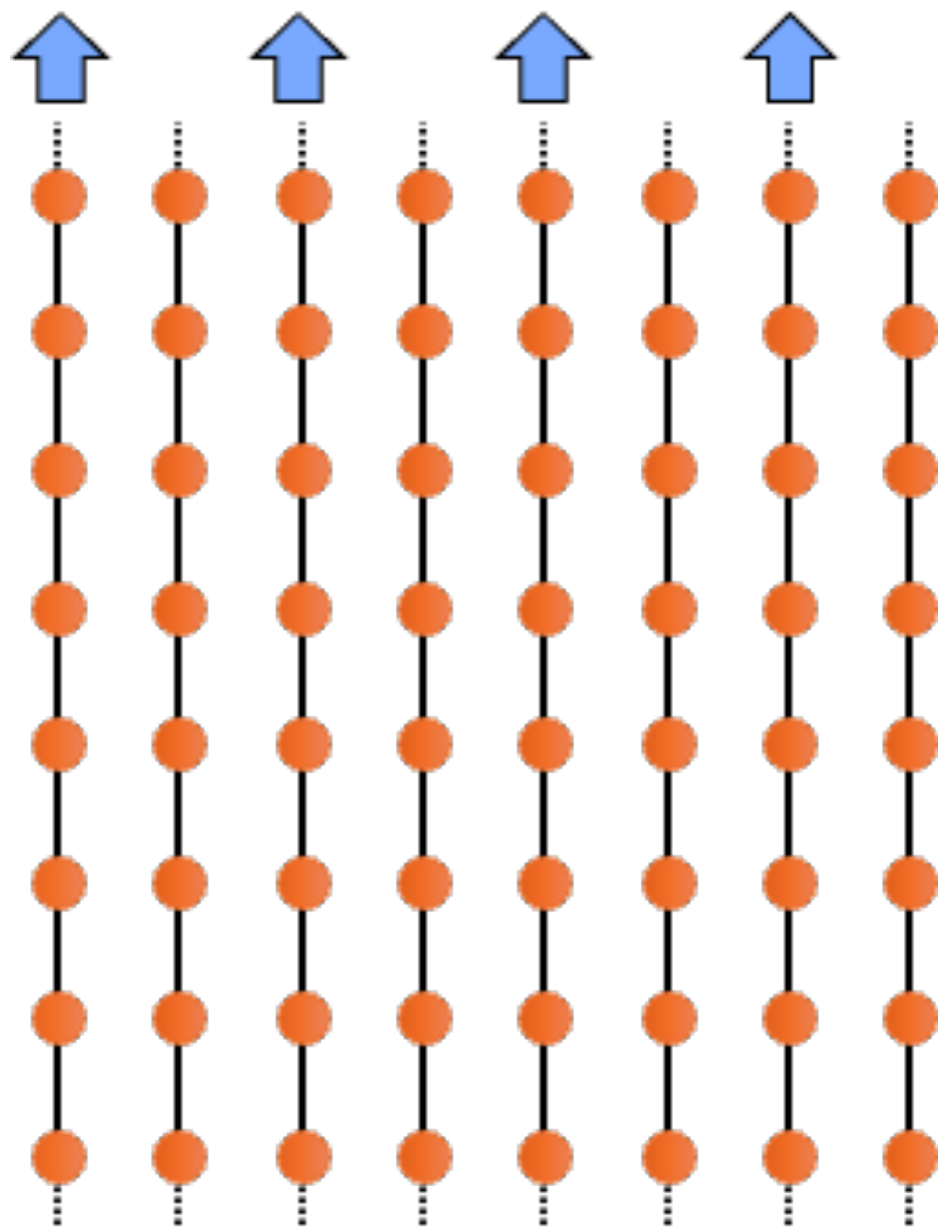

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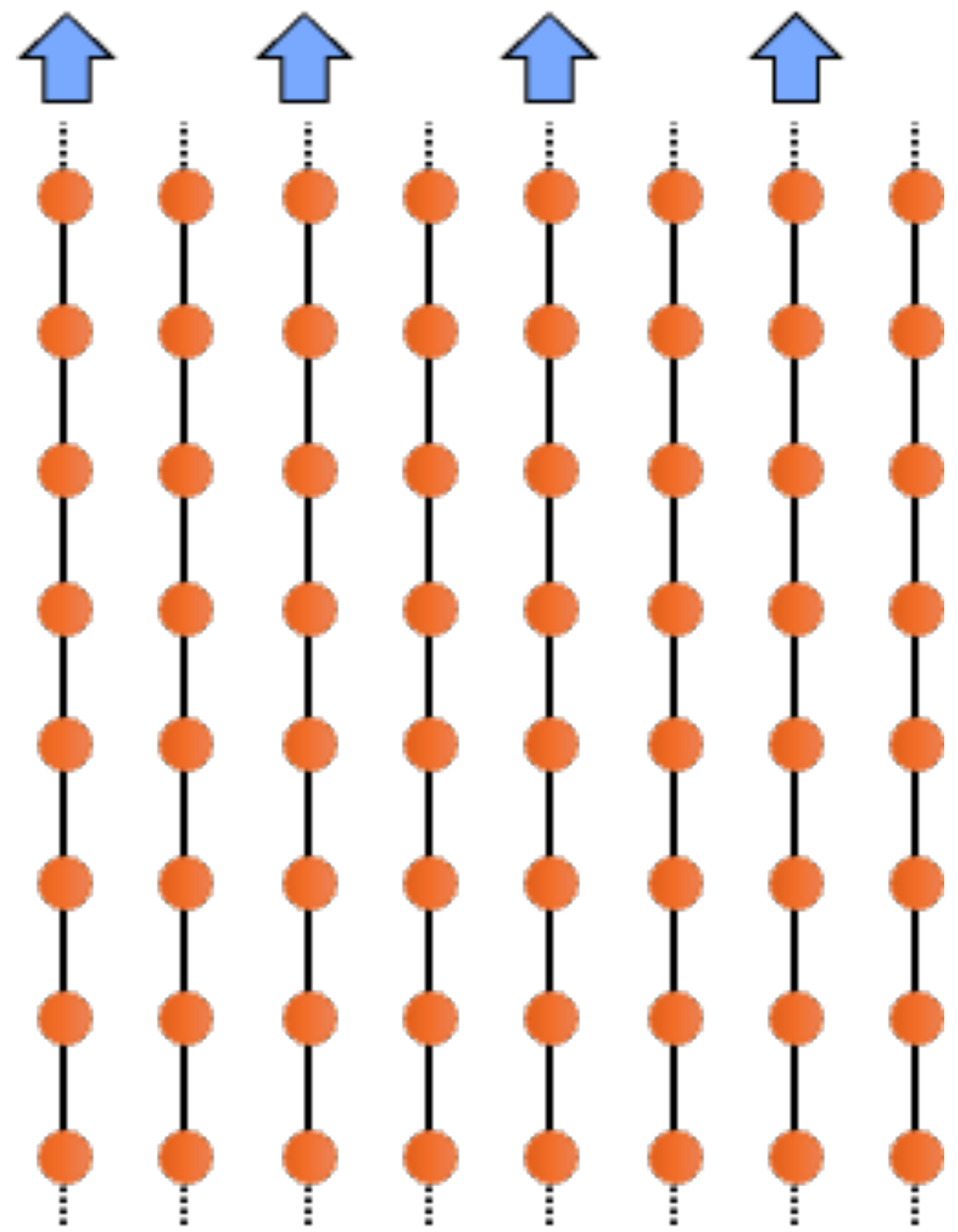

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Observable of interest:

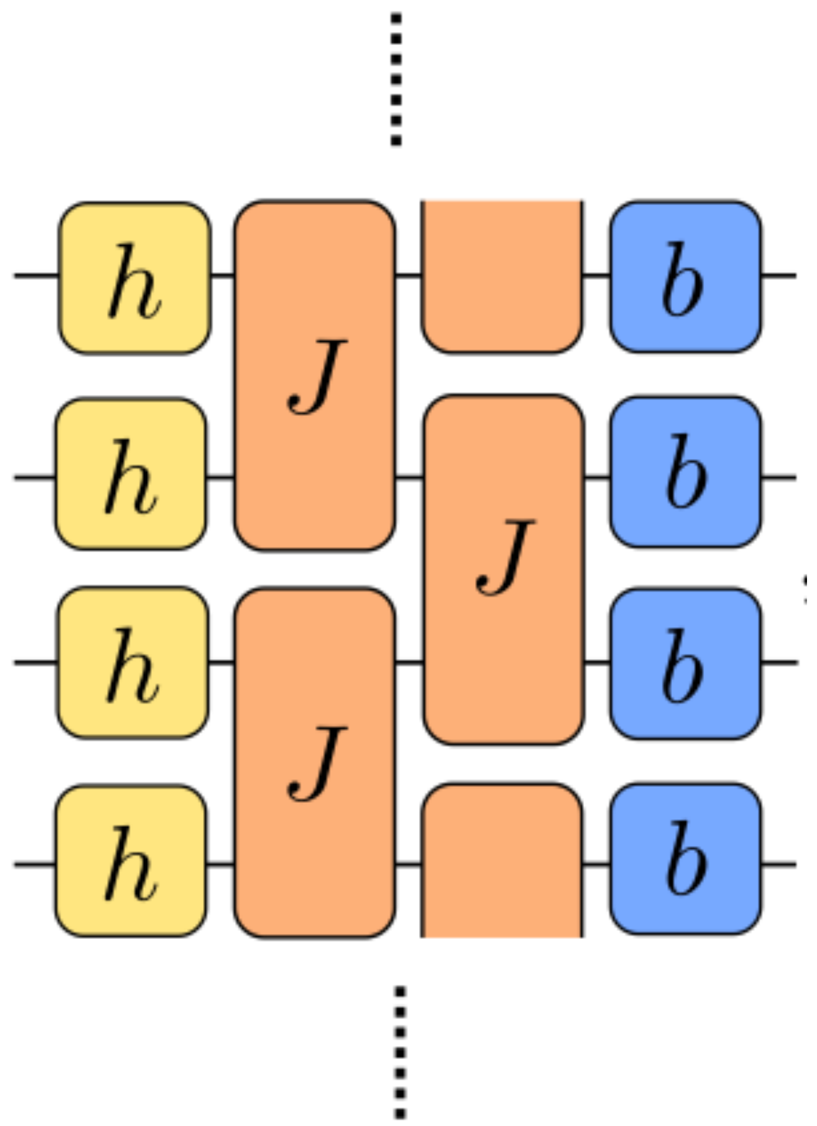
Infinite temperature autocorrelation function: $C_n(t) = Tr[\hat{\rho}_\infty \hat{X}_0(0)\hat{X}_n(t)]$, $\hat{\rho}_\infty = \frac{\mathrm{I}}{2^N}$

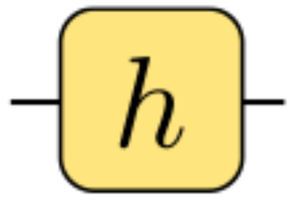
Circuit components

Floquet unitaries implemented as two qubit gates in a brickwork layout.

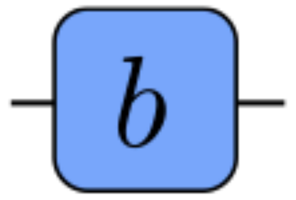
Floquet Unitary:

$$U_{KI} = e^{-iH_K} e^{-iH_I} \rightarrow e^{-ib\sum \sigma^x} e^{-iJ\sum \sigma^z \sigma^z} e^{-ih\sigma^z}$$





$= e^{-ih\sigma^z}$



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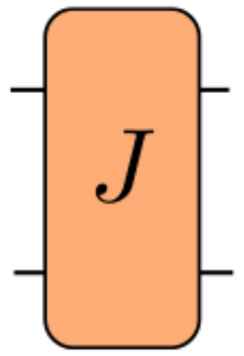

$= e^{-iJ\sigma^z \otimes \sigma^z}$

Circuit components

Floquet unitaries implemented as two qubit gates in a brickwork layout.

Floquet Unitary:

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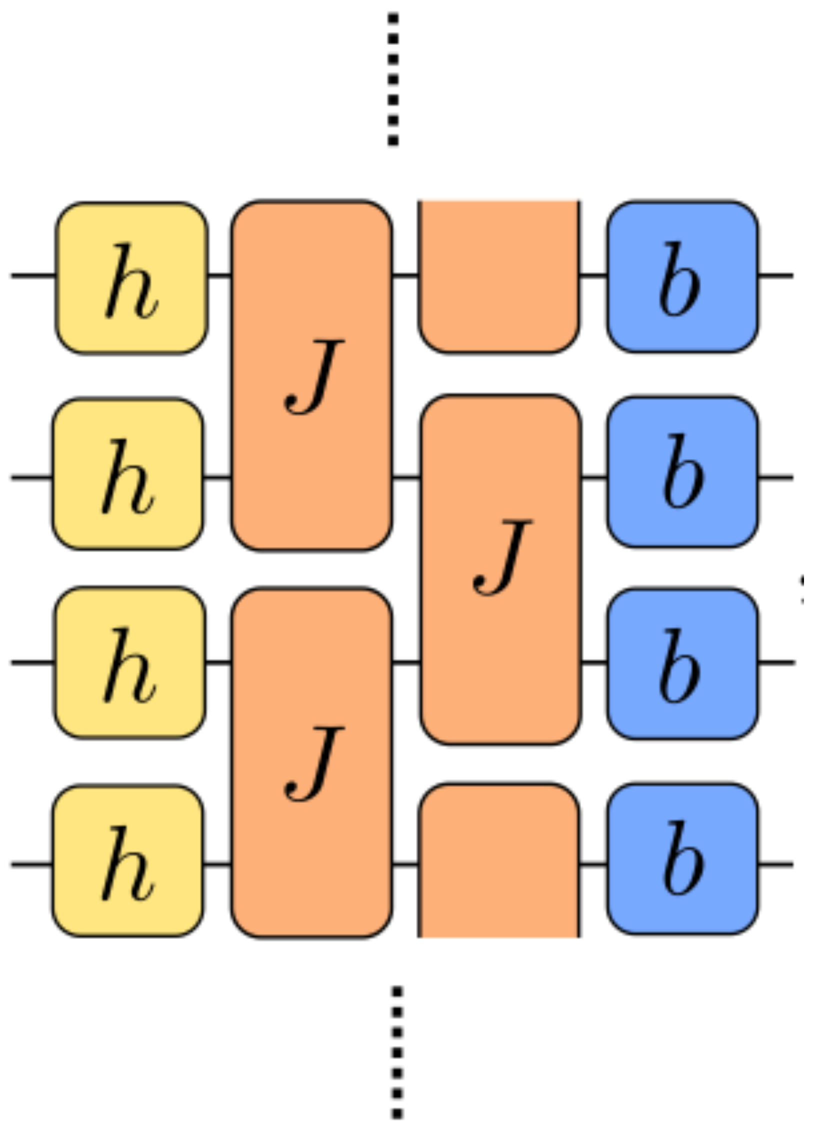
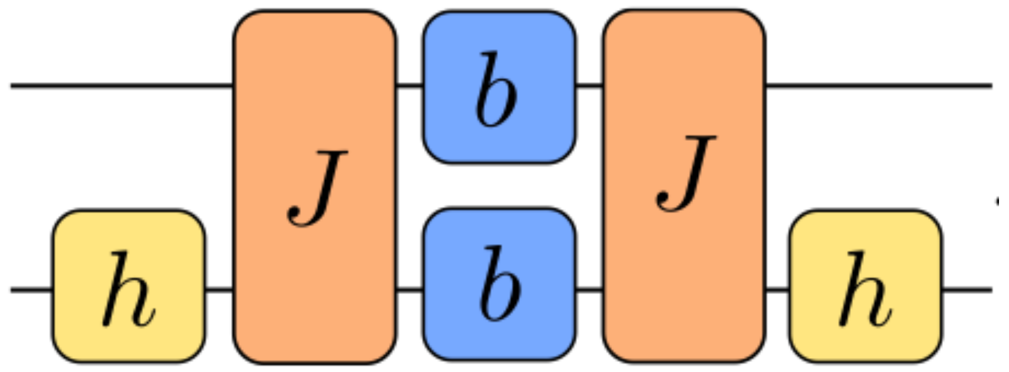
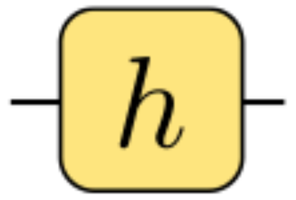
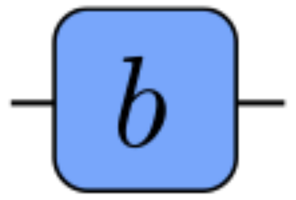


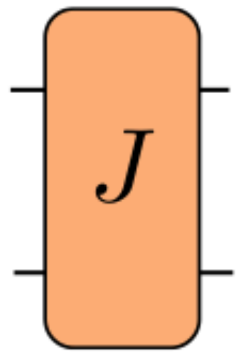
$U_{n,n+1} =$ 



$= e^{-ih\sigma^z}$



$= e^{-ib\sigma^x}$



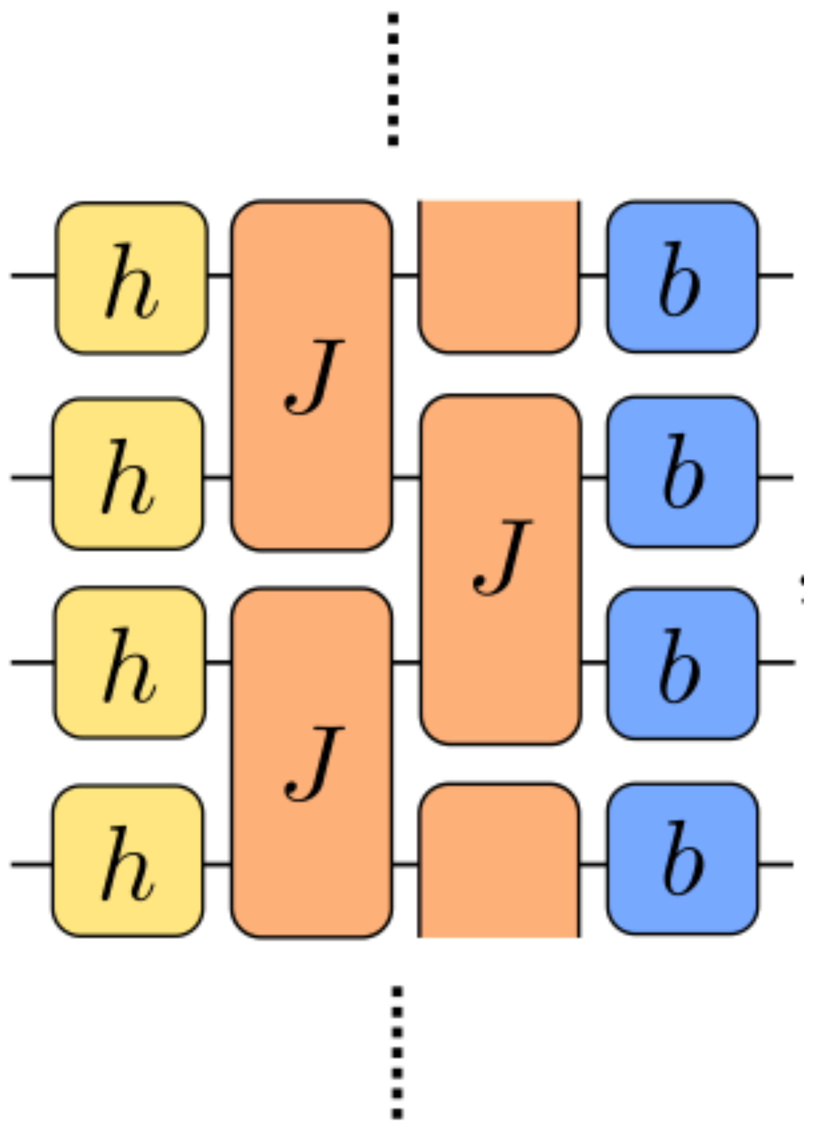
$= e^{-iJ\sigma^z \otimes \sigma^z}$

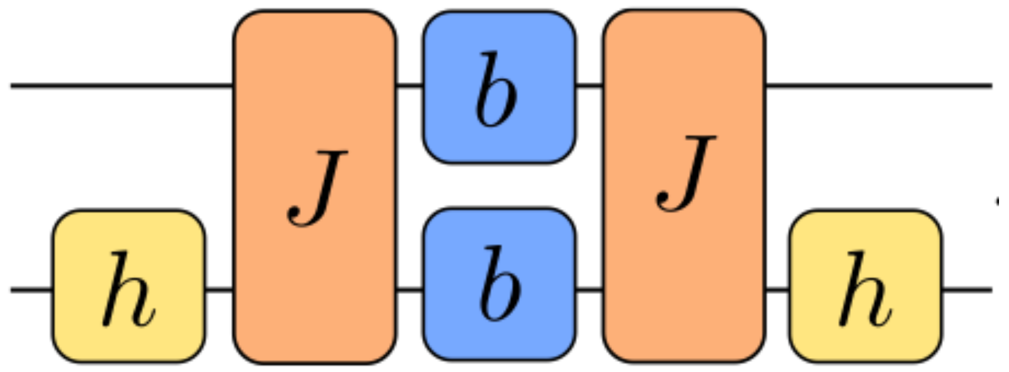
Circuit components

Floquet unitaries implemented as two qubit gates in a brickwork layout.

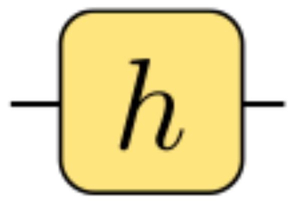
Floquet Unitary:

$$U_{KI} = e^{-iH_K} e^{-iH_I} \rightarrow e^{-ib\sum \sigma^x} e^{-iJ\sum \sigma^z\sigma^z} e^{-ih\sigma^z}$$

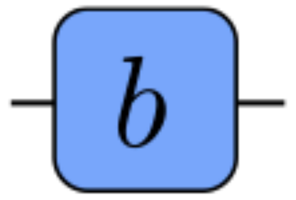


$U_{n,n+1} =$ 

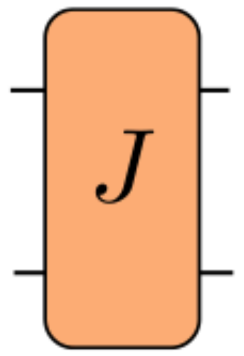
Dual unitary for $J = b = \frac{\pi}{4}$



$= e^{-ih\sigma^z}$



$= e^{-ib\sigma^x}$



$= e^{-iJ\sigma^z \otimes \sigma^z}$

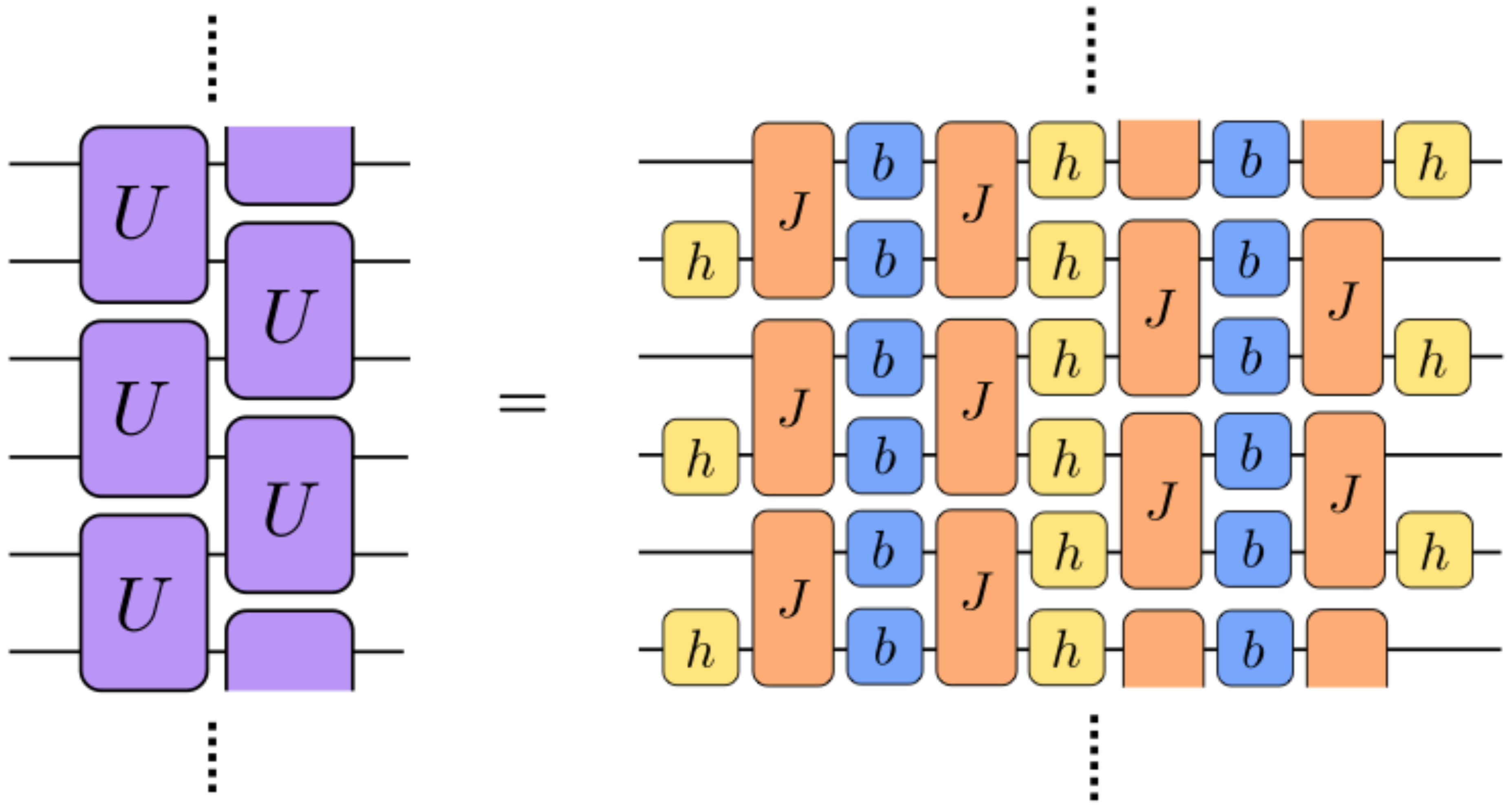
Circuit components

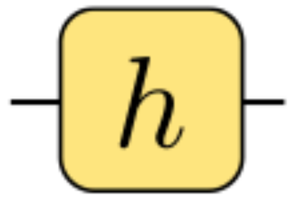
Floquet unitaries implemented as two qubit gates in a brickwork layout.

$$U_{even} = \prod_{n_{even}} U_{n,n+1} \qquad U_{odd} = \prod_{n_{odd}} U_{n,n+1}$$

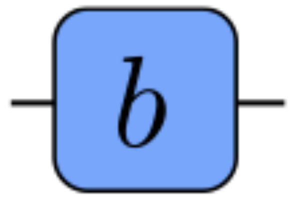
One time step:

$U_1 = U_{even} U_{odd} \rightarrow$

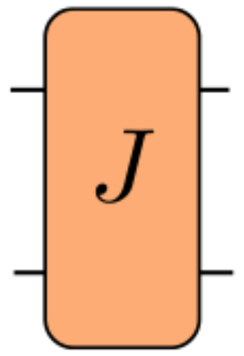




$= e^{-ih\sigma^z}$



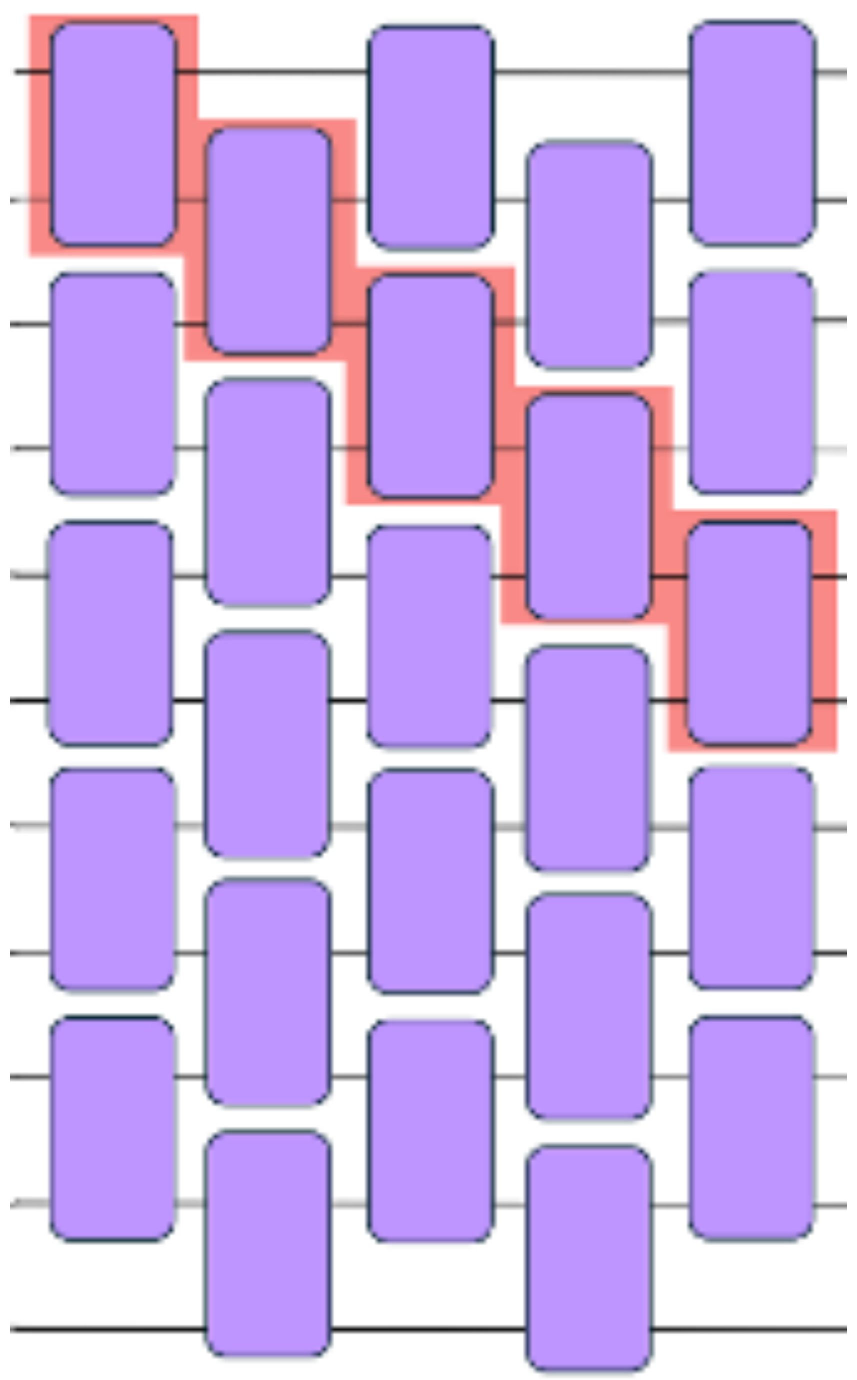
$= e^{-ib\sigma^x}$

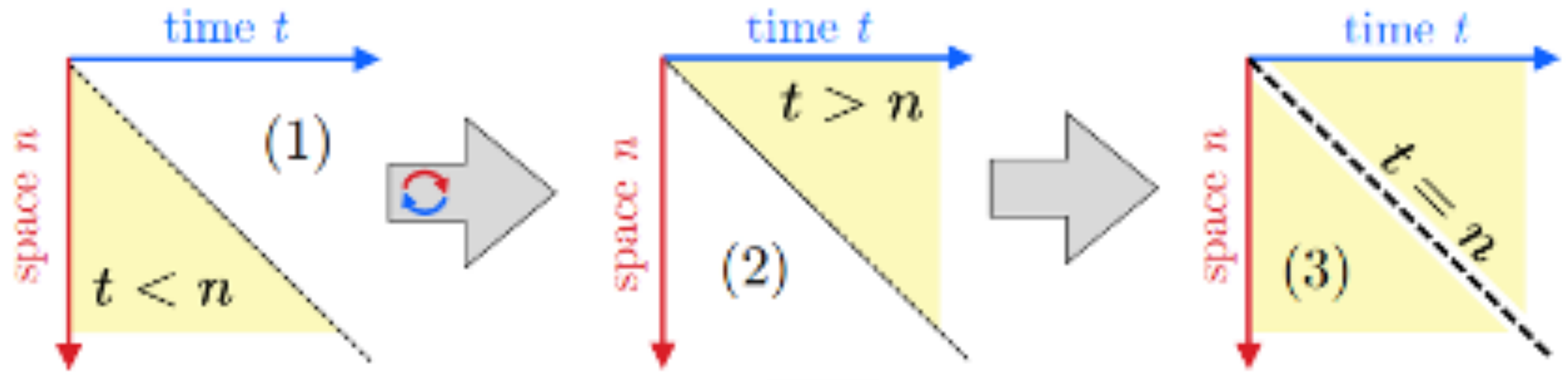


$= e^{-iJ\sigma^z \otimes \sigma^z}$

Dual unitary

For dual unitary brickwork circuits the signal will propagate along the light cone

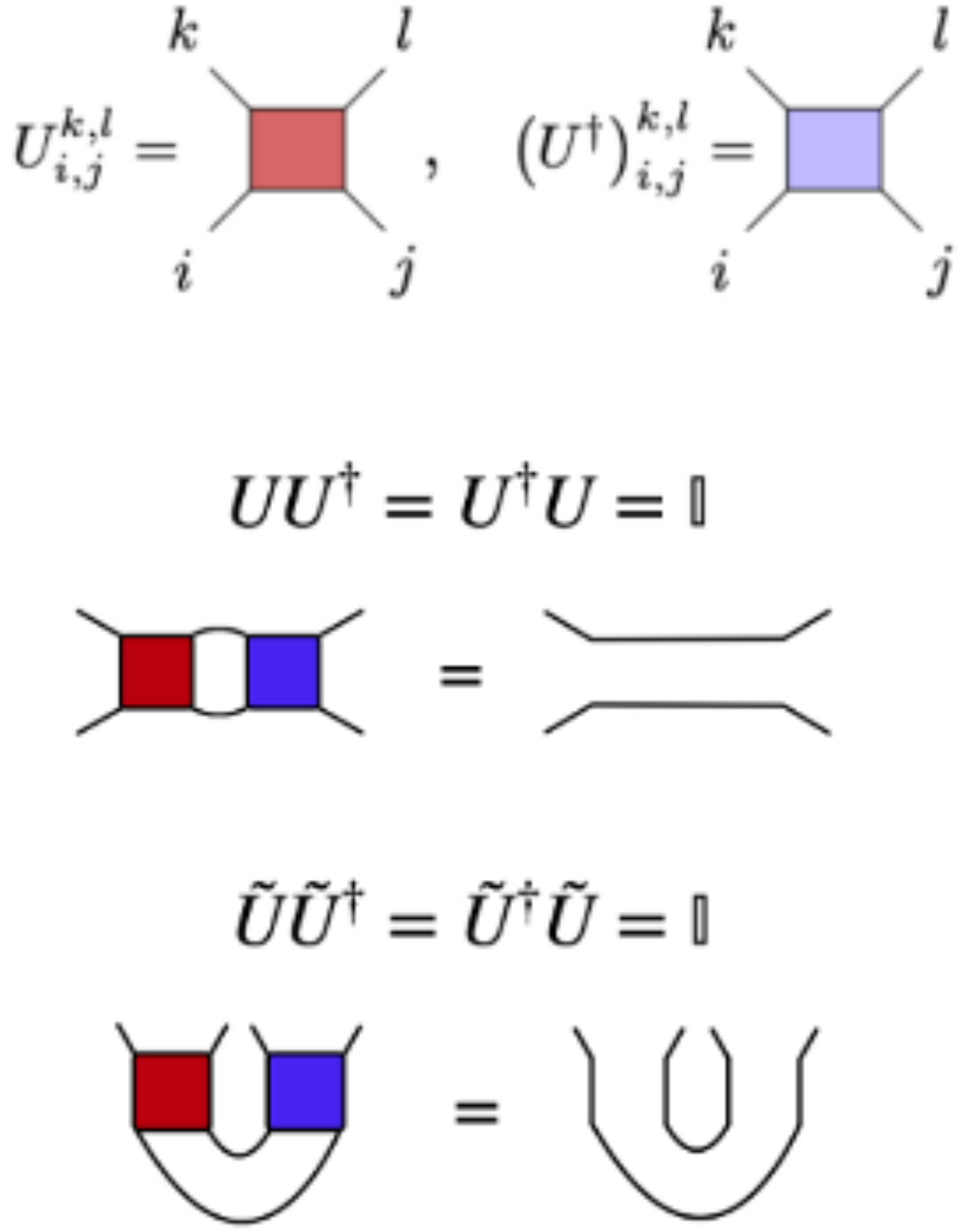




Information spreading for any brickwork circuit: C(t) = 0 for t<n

Signal propagates along the light cone only

Dual unitary circuits limit information spread in the spatial direction C(t) = 0 for t>n



Three regimes

Three regimes

1

$$J = b = \frac{\pi}{4}, \qquad h = 0$$

Integrable

Clifford Gates

Exact solution:

$$C(t) = \begin{cases} 1, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$$

Dual Unitary

Three regimes

1

$$J = b = \frac{\pi}{4}, \qquad h = 0$$

Integrable

Clifford Gates

Exact solution:

$$C(t) = \begin{cases} 1, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$$

2

$$J = b = \frac{\pi}{4}, \qquad h \neq 0$$

Non-integrable

Non-Clifford

Exact solution:

$$C(t) = \begin{cases} [cos(2h)]^t, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$$

Dual Unitary

Three regimes

1

$$J = b = \frac{\pi}{4}, \qquad h = 0$$

Integrable

Clifford Gates

Exact solution:

$$C(t) = \begin{cases} 1, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$$

2

$$J = b = \frac{\pi}{4}, \qquad h \neq 0$$

Non-integrable

Non-Clifford

Exact solution:

$$C(t) = \begin{cases} [cos(2h)]^t, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$$

3

$$J = \frac{\pi}{4}, \qquad b \neq \frac{\pi}{4}, \qquad h \neq 0$$

Non-integrable

Non-Clifford

No exact solution

Dual Unitary (1, 2)

Non dual unitary (3)

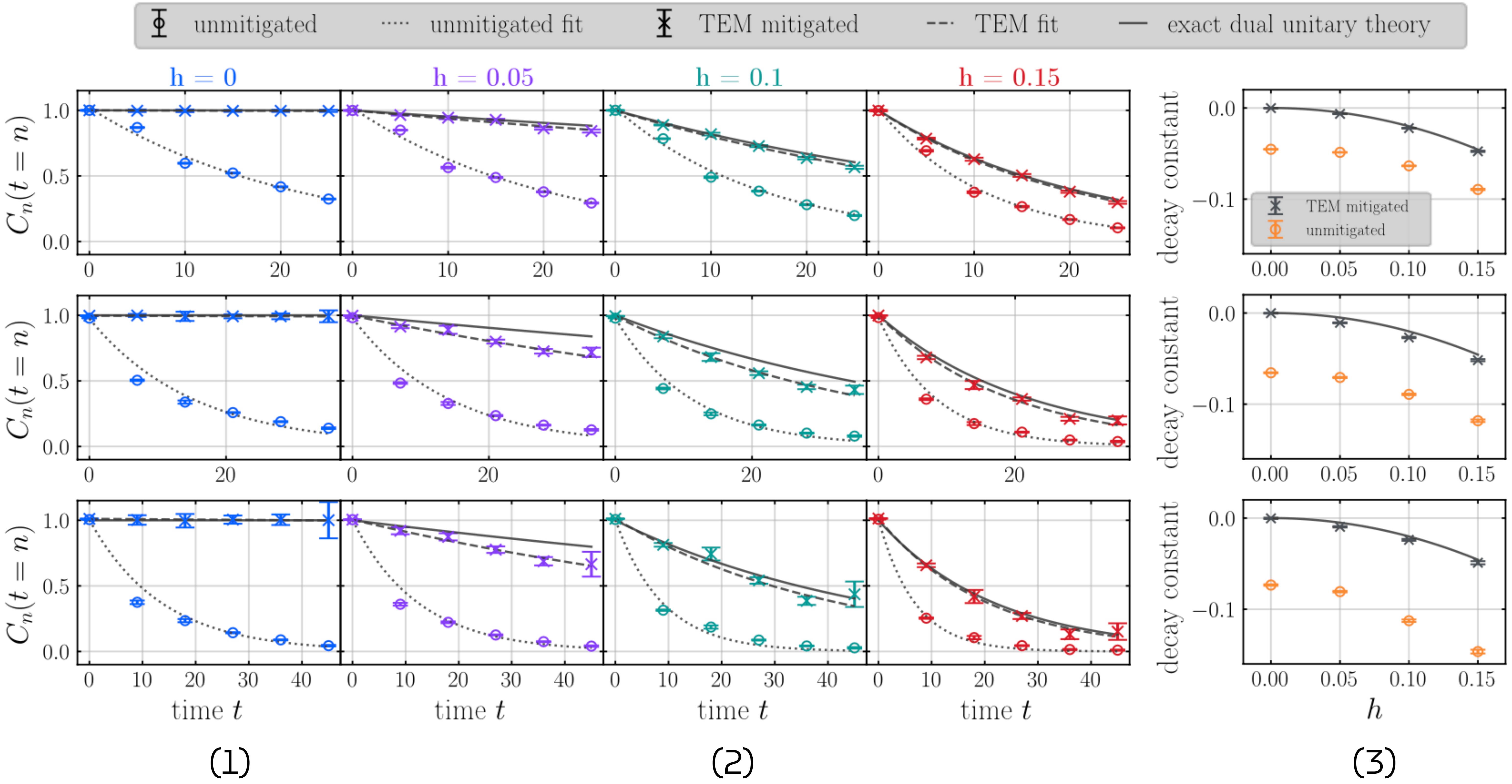
Experimental results run on IBM Eagle

Autocorrelation function at the dual unitary point

2 Dual Unitary: b = J = $\pi/4$

Non-integrable

Exact solution: $C(t) = \begin{cases} [cos(2h)]^t, & \text{if } t = n \\ 0, & \text{if } otherwise \end{cases}$



(1)

- Clifford for h=0. Used to calibrate the noise model parameters adjusted to fit the mitigated curve.

(2)

- TEM mitigated results match the analytical decay for varying system sizes.
- Imperfections directly linked to imperfections in noise characterisation.

(3)

- Validation: decay rates of the mitigated results match theory

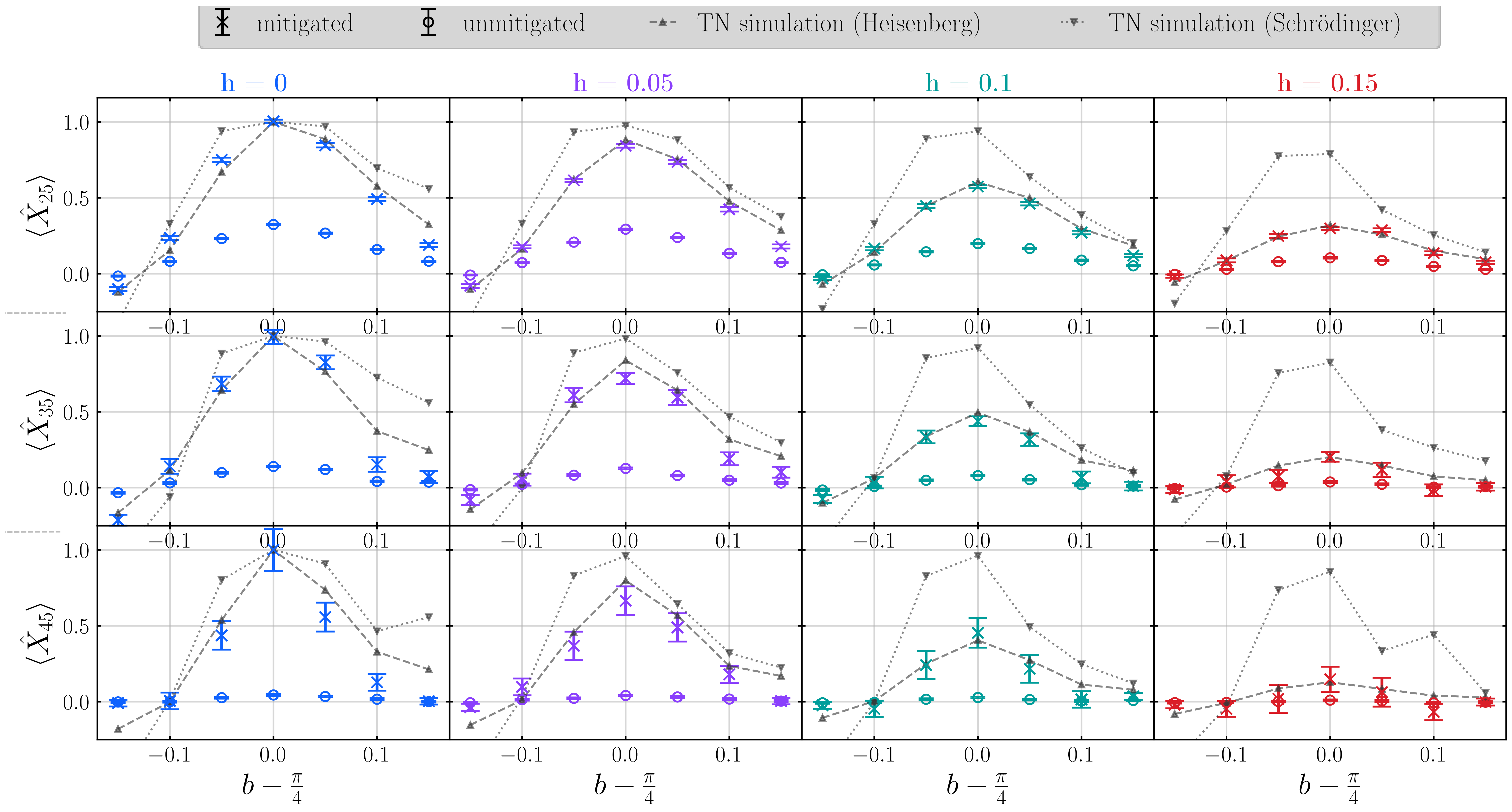
Moving away from the dual unitary point

3

Not dual unitary

Non-integrable

No exact solution



No analytical solution exists nor brute force solution so therefore we must compare different methods for simulation:

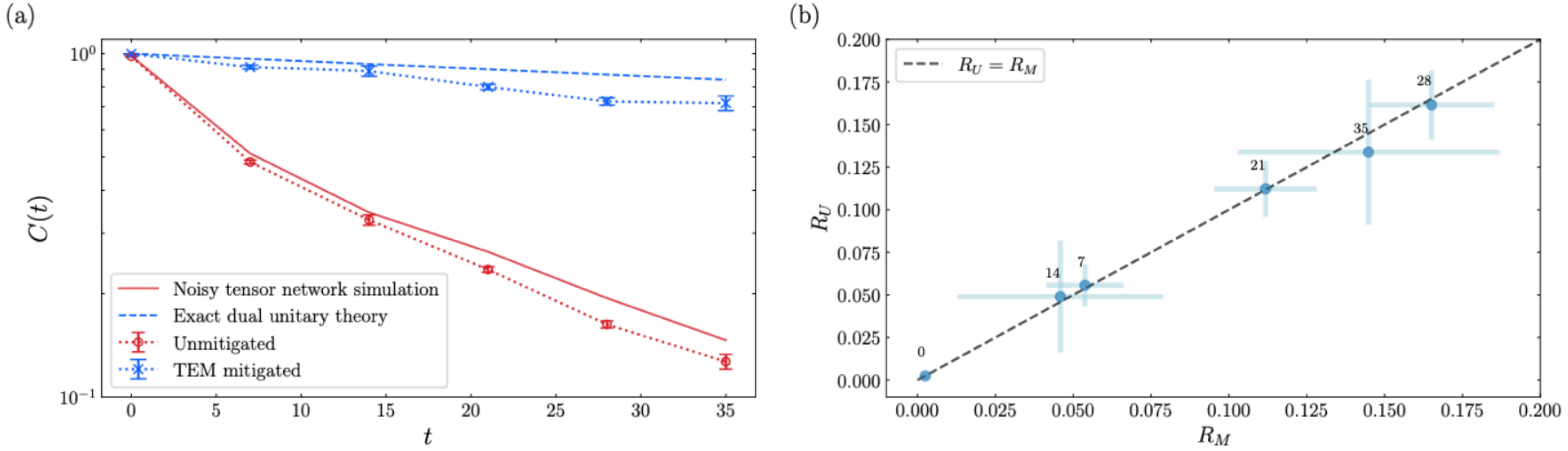
- Quantum + TEM
- TN Schrödinger
- TN Heisenberg

Accurate recovery of near zero signal that is indistinguishable from background statistical noise

Computing expectation values $\langle \hat{X}_t(t) \rangle$ for $t = (N-1)/2$

Impact of noise model discrepancies

We are only as good as our noise characterisation



- Tensor Network simulations using the noise model provided can show us the accuracy of the model when compared to the noisy signal obtained from hardware.
- Where there is a mismatch in noisy signal to noisy simulation, there will be a comparable mismatch between the TEM result and the ideal curve.

71 qubit dual unitary experiment, h = 0.05

Sampling overhead

N_{qubits}	R	$\Gamma_{\text{PEC}}/\Gamma_{\text{TEM}}$	$\Gamma_{\text{ZNE}}/\Gamma_{\text{TEM}}$
51	3.1	9.6	25.6
71	7.1	50.4	64.6
91	22.7	515	149

When we are considering system sizes where the numbers of shots are in the tens of millions, these factors are prohibitive.

Sampling overhead

Fix

$$\Gamma \sim e^{\epsilon N L}$$

Exponent blows up for fixed error rate as circuit area increases while it gets easier to simulate classically as everything approaches the maximally mixed state.

Sampling overhead

Fix

$$\lim_{\epsilon \to 0} \Gamma \sim e^{\epsilon N L}$$

Larger circuit sizes are enabled as hardware improves.

Quantum + EM becomes favorable as things get more difficult to simulate classically.

Noise agnostic error mitigation for specific problems (ground state simulation) may be a viable alternative.

This could be accessible for complex circuits without repetition (Chemistry) where noise learning would be prohibitively hard

Could be combined with intermediate-scale QEC to mitigate the residual errors

Conclusion

Noise remains a prominent challenge to overcome and development of new methods for noise mitigation will have critical impact in the evolution of the field.

Combining quantum computing with HPC is advantageous for increasing the reach of error mitigation methods.

The results shown highlight the utility of quantum simulation, even on pre-fault tolerant devices for studying models of physical interest

TEM can be advantageous with respect to purely classical tensor network methods, given that the tensor network in TEM does not need to account for the state of the quantum computer, nor the evolved observable in the Heisenberg picture

Instead, the tensor network represents the inverse of the noise channel in the quantum processor, which approaches identity for decreasing noise.

Therefore, the classical computational complexity needed by TEM also decreases, hence enabling us to obtain accurate results with smaller computational cost than a classical-only tensor network approach.

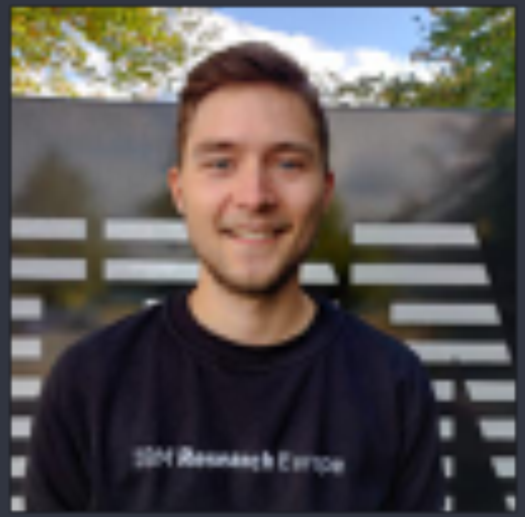
A rethinking of the way we do things is necessary to discover how best to combine quantum error mitigation with error correction

algorithmiq

IBM

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin




Laurin Fischer
Andrew Eddins
Francesco Tacchino
Andre He
Youngseok Kim
Ivano Tavernelli
Abhinav Kandala

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Davide Ferracin
Francesca Pietracaprina
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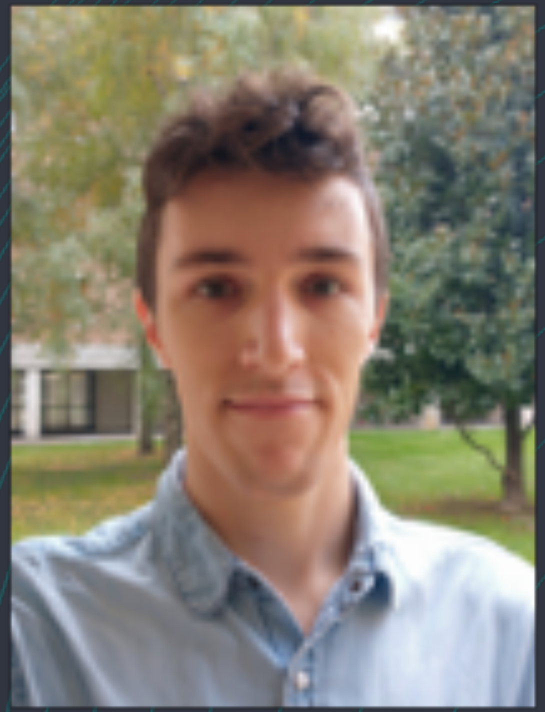



Thank you!