IQuS InQubator for Quantum Simulation

UNIVERSITY of WASHINGTON

Quantum Simulation of Adiabatic State Preparation

Z. Li, D. M. Grabowska, and M. J. Savage, (2024), arXiv:2407.13835 [quant-ph].

Zhiyao Li

University of Washington

WERQSHOP 07/17/2025

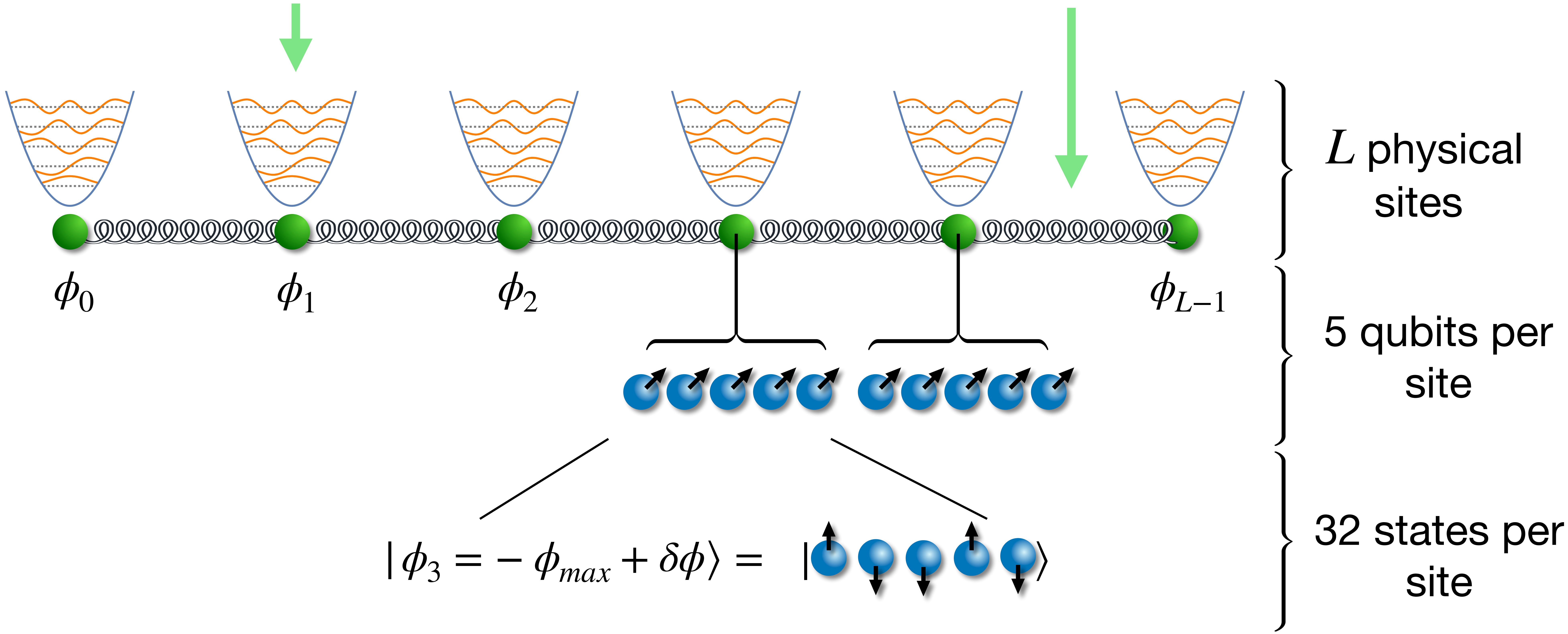
M. J. Savage

D. M. Grabowska

$\lambda\phi^4$ scalar field theory

$$\frac{1}{2}\hat{\Pi}_k^2 + \frac{1}{2}m^2\hat{\phi}_k^2 + \frac{\lambda}{4!}\hat{\phi}_k^4$$

$$\frac{1}{2}\left(\hat{\phi}_k - \hat{\phi}_{k+1}\right)^2$$



N. Klco and M. J. Savage, Phys. Rev. A 99, 052335 (2019), arXiv:1808.10378 [quant-ph]

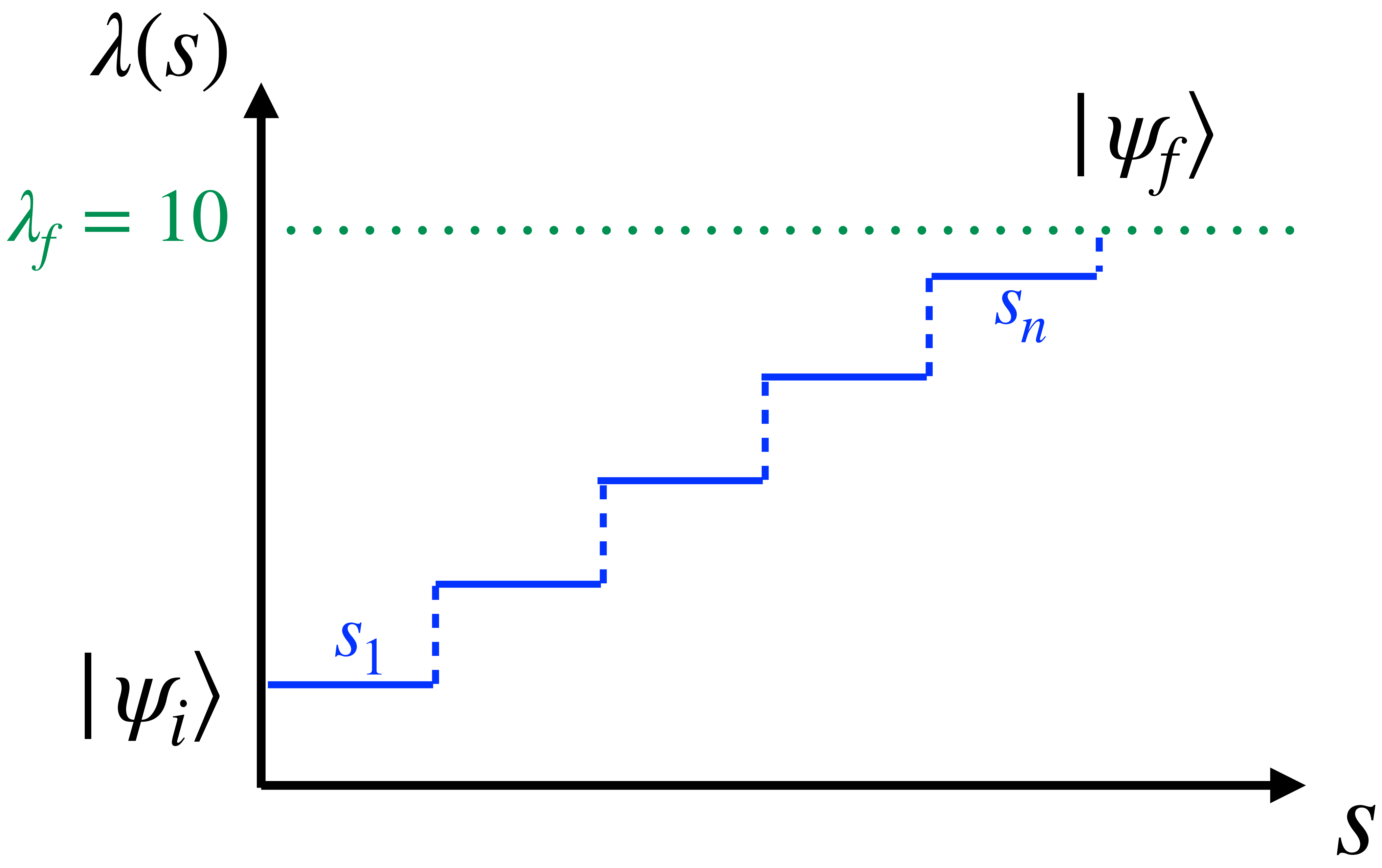
N. A. Zemlevskiy, (2024), arXiv:2411.02486 [quant-ph]

Adiabatic State Preparation

$$\hat{H}^{\text{latt.}} = \sum_k \frac{1}{2}\hat{\Pi}_k^2 + \frac{1}{2}m^2\hat{\phi}_k^2 + \frac{1}{2}\left(\hat{\phi}_k - \hat{\phi}_{k+1}\right)^2 + \boxed{\frac{\lambda}{4!}\hat{\phi}_k^4}$$

Slowly turn on interaction

$$|\psi_1\rangle = e^{-i\hat{H}(s_1)\delta t}|\psi_i\rangle$$



S. P. Jordan, K. S. M. Lee, and J. Preskill, Quant. Inf. Comput. (2014)

Quantum Simulation

Digitization → Map the scalar field onto finite Hilbert space

N. Klco and M. J. Savage, Phys. Rev. A 99, 052335 (2019)

C. W. Bauer and D. M. Grabowska, Phys. Rev. D 107, L031503 (2023)

Circuit Construction → Free Theory State Preparation

M. Möttönen, J. J. Vartiainen, V. Bergholm, and M. M. Salomaa, Physical Review Letters 93 (2004)

N. Klco and M. J. Savage, Phys. Rev. A 102, 012612 (2020)

Circuit Construction → Adiabatic Time Evolution

Error Mitigation → Pauli Twirling, Dynamical Decoupling, TREX

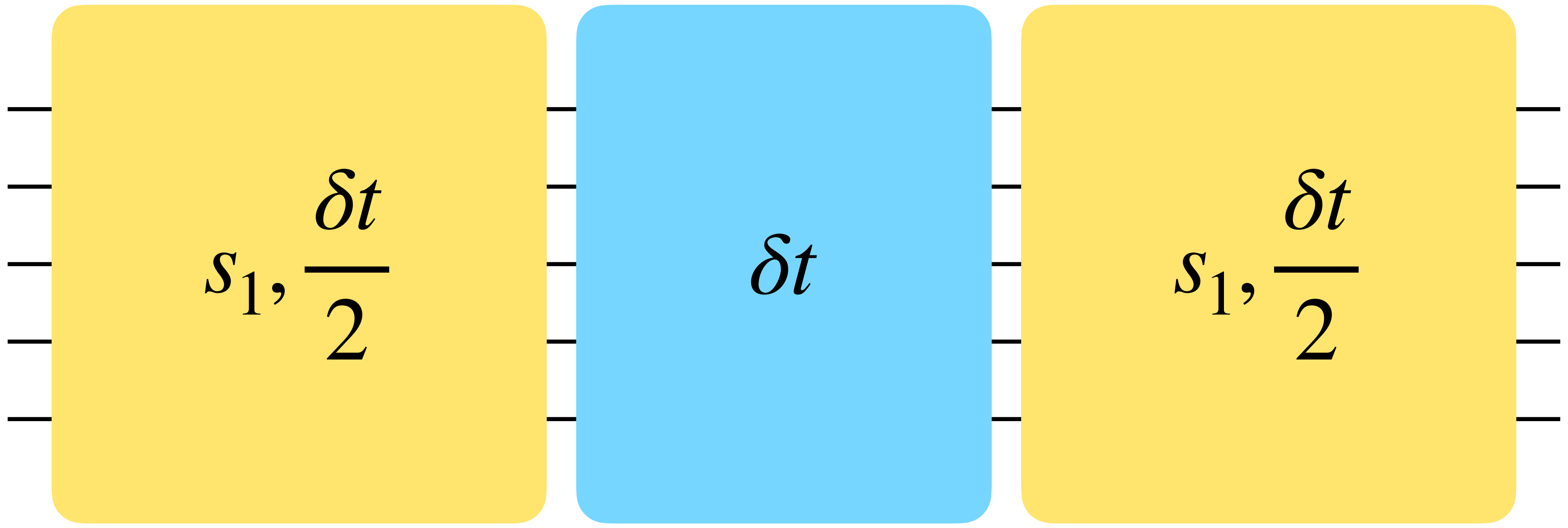
Error Mitigation → Decoherence Renormalization

Quantum Circuits

Error Mitigation: Decoherence Renormalization

$$\hat{H} = \frac{1}{2}\hat{\Pi}^2 + \frac{1}{2}m^2\hat{\phi}^2 + \frac{\lambda(s)}{4!}\hat{\phi}^4$$

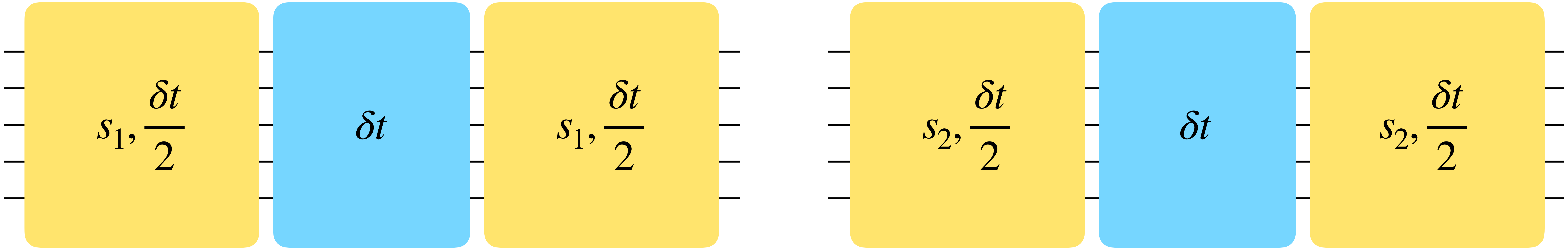
$$\tilde{\Phi}(s,t) \equiv e^{-i(\frac{1}{2}\hat{\phi}^2 + \frac{\lambda(s)}{4!}\hat{\phi}^4)t} \quad \text{and} \quad \tilde{\Pi}(t) \equiv e^{-i\frac{1}{2}\hat{\Pi}^2 t}$$



Quantum Circuits

Error Mitigation: Decoherence Renormalization

$$\hat{H} = \frac{1}{2}\hat{\Pi}^2 + \frac{1}{2}m^2\hat{\phi}^2 + \frac{\lambda(s)}{4!}\hat{\phi}^4 \qquad \tilde{\Phi}(s,t) \equiv e^{-i\left(\frac{1}{2}\hat{\phi}^2 + \frac{\lambda(s)}{4!}\hat{\phi}^4\right)t} \quad \text{and} \quad \tilde{\Pi}(t) \equiv e^{-i\frac{1}{2}\hat{\Pi}^2 t}$$



Quantum Circuits

Error Mitigation: Decoherence Renormalization

$$\hat{H} = \frac{1}{2}\hat{\Pi}^2 + \frac{1}{2}m^2\hat{\phi}^2 + \frac{\lambda(s)}{4!}\hat{\phi}^4$$

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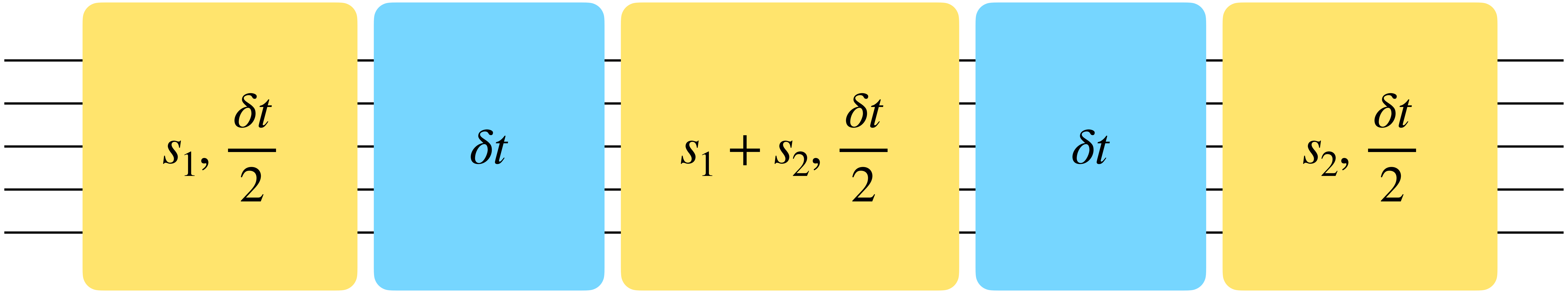

Quantum Circuits

Error Mitigation: Decoherence Renormalization

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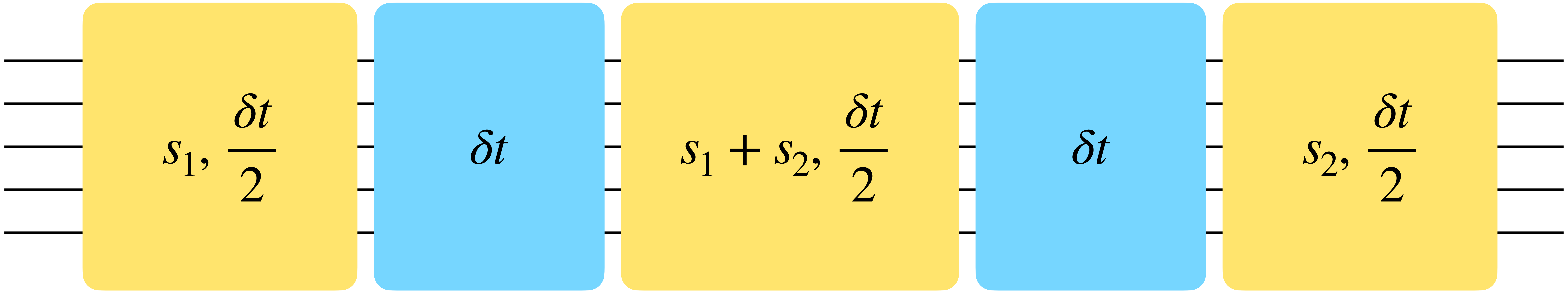
Physics Circuit



Quantum Circuits

Error Mitigation: Decoherence Renormalization

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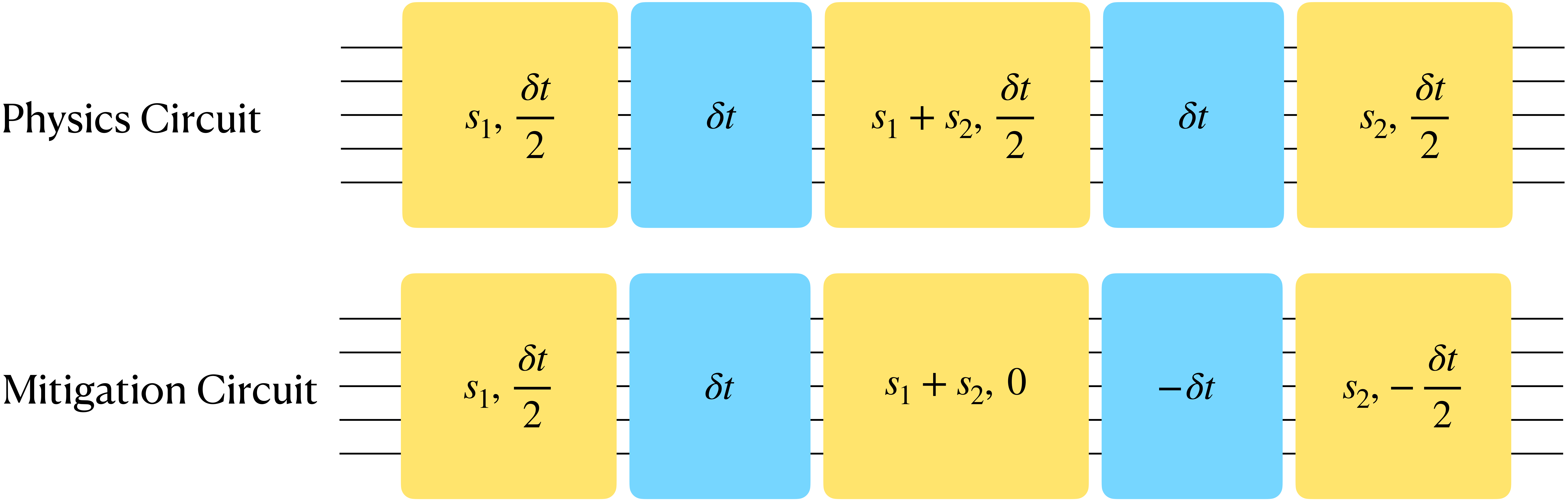
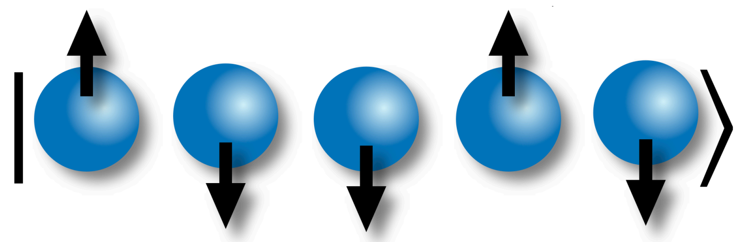


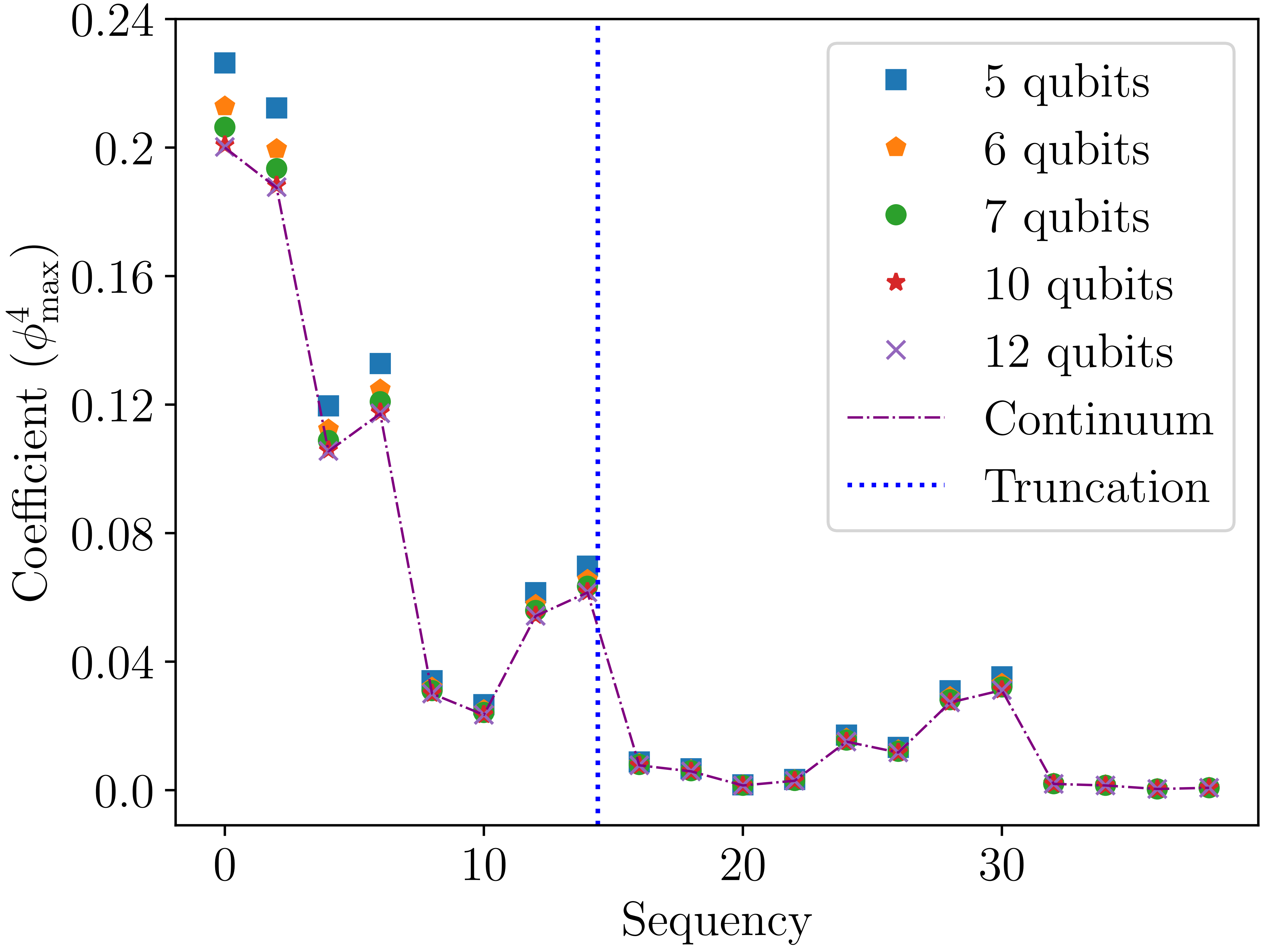
Sequency Hierarchy Truncation

$$\hat{\mathcal{O}} = \sum_{\nu=0}^{2^{n_q}-1} \beta_\nu \hat{\mathcal{O}}_\nu \quad \rightarrow \quad e^{-i\hat{\mathcal{O}}t} = \prod_{\nu=0}^{2^{n_q}-1} e^{-i\beta_\nu \hat{\mathcal{O}}_\nu t}$$

- Decompose (band)diagonal operators into sequency basis
- Digital counterpart of the Fourier series (loosely)
- Hierarchy in sequency coefficients
- Connected to qubit hierarchy



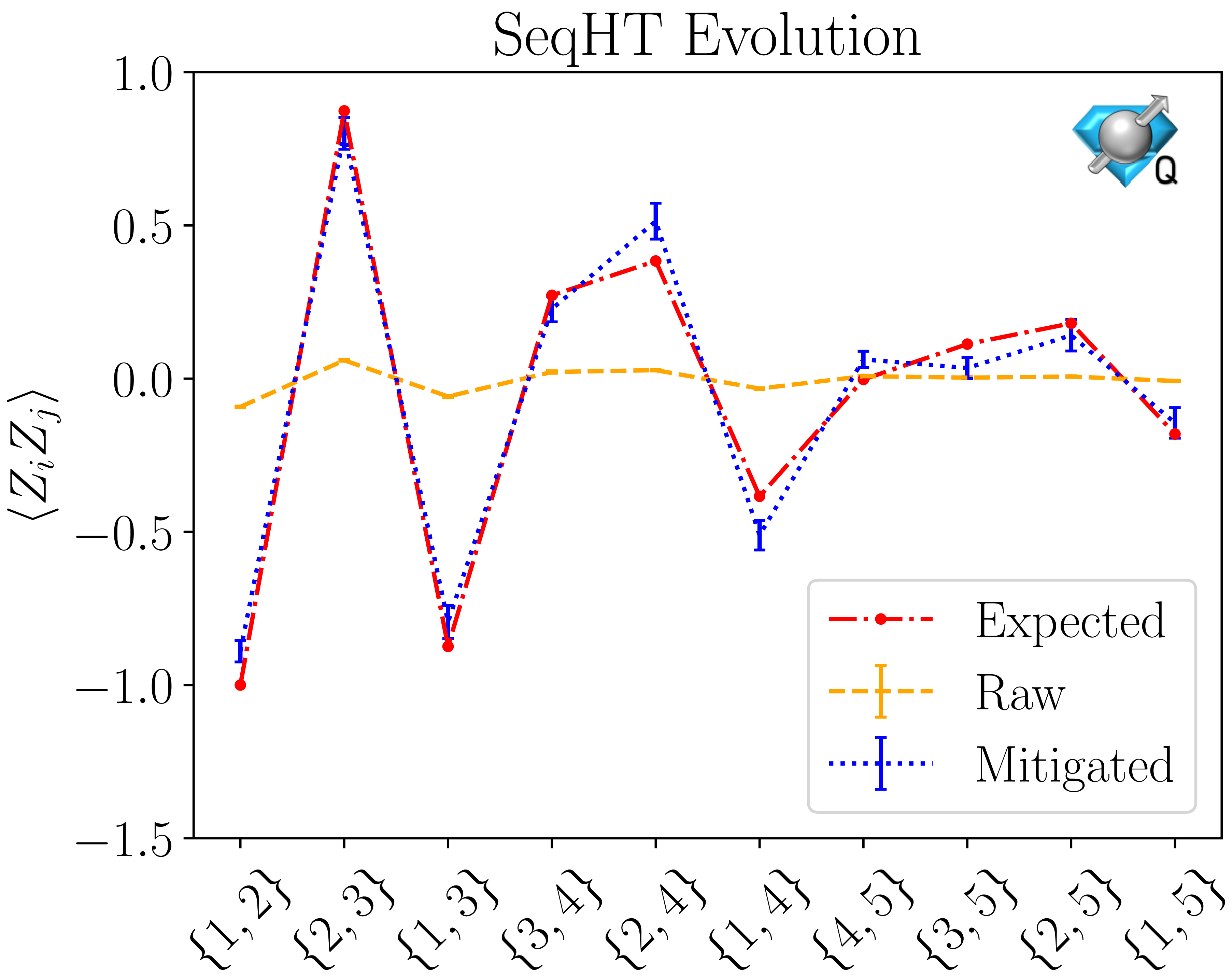
SeqHT of $\hat{\phi}^4$



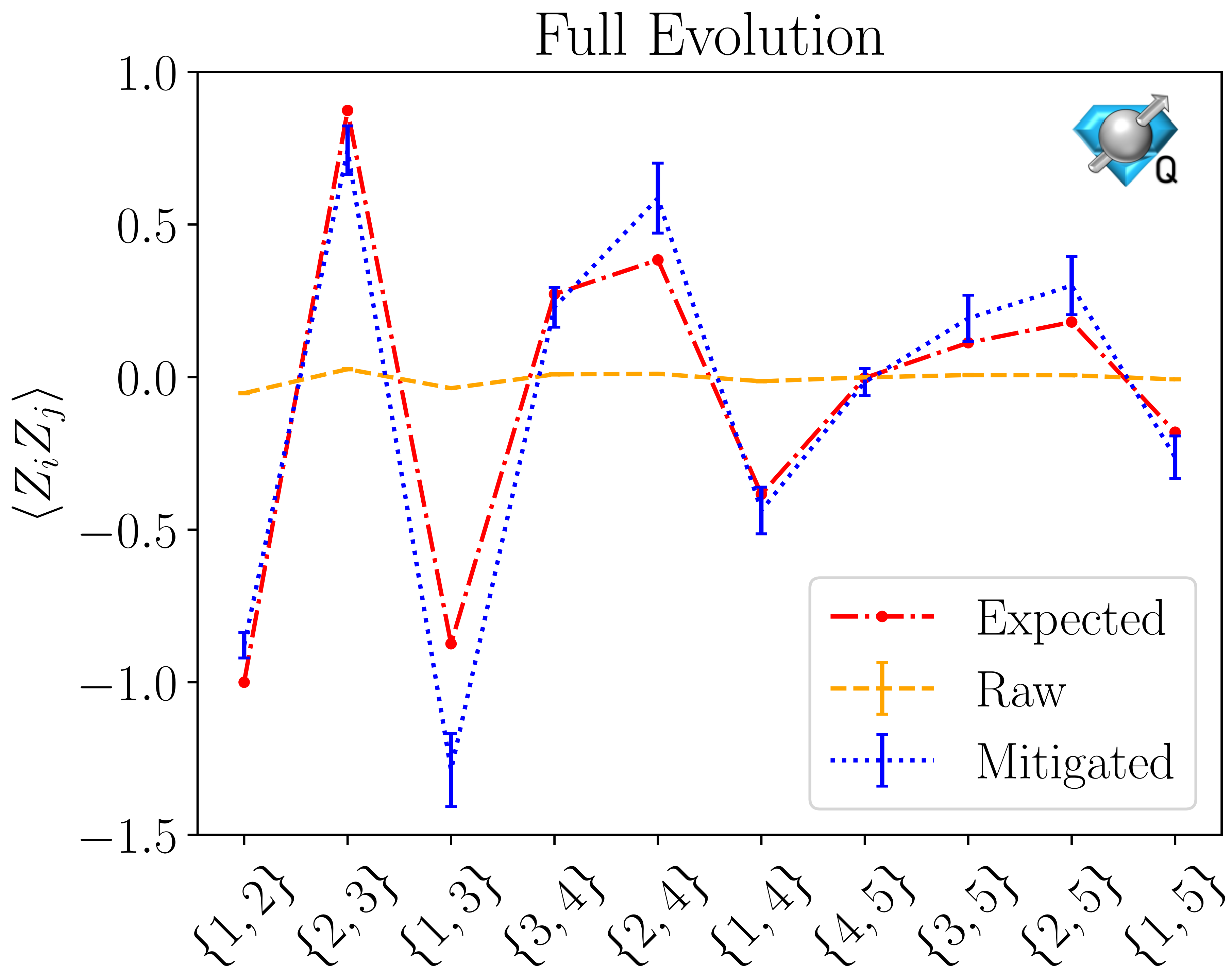
Quantum Simulations with SeqHT

on IBM's Quantum Computer `ibm_sherbrooke`

Expectation values of ZZ operators in the ground state of $\lambda\phi^4$ theory with $\phi_{max} = 4$ and $\lambda = 10$



237 two-qubit gates
with depth 208



321 two-qubit gates
with depth 291

$n_q = 5$

$\nu_{cut} = 14$

80 Pauli twirls

8000 shots per twirl

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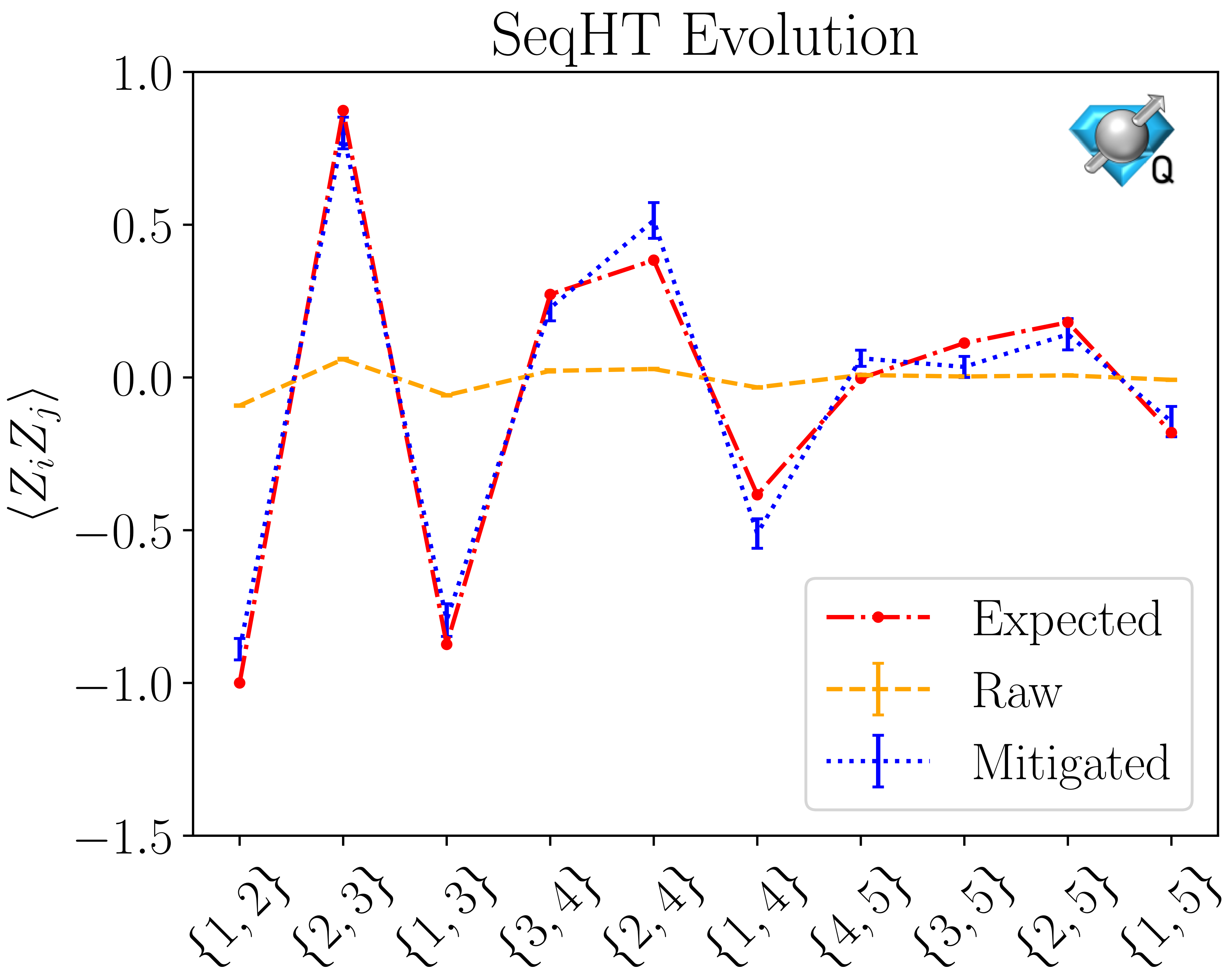

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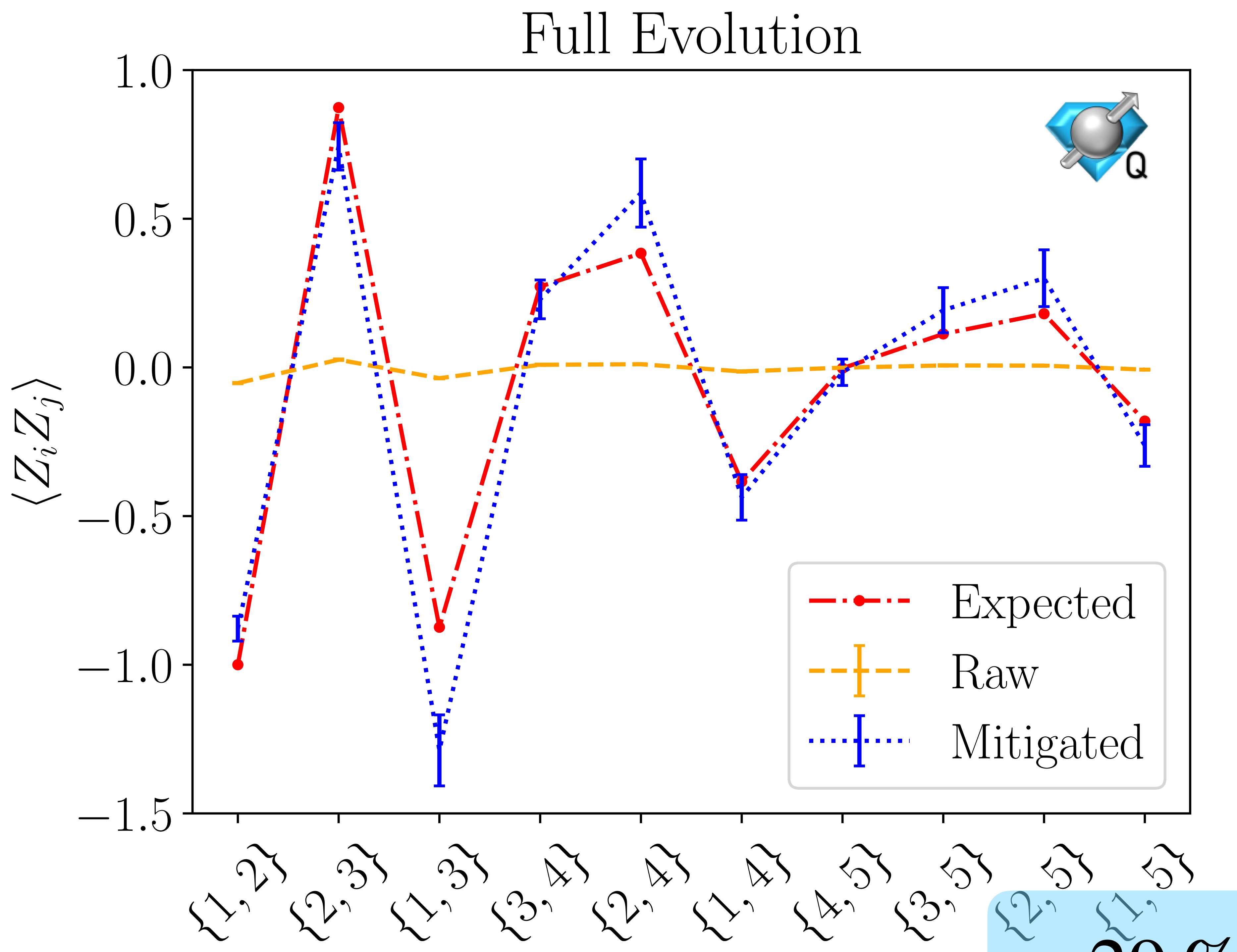


321 two-qubit gates
with depth 291

$n_q = 5$

$\nu_{cut} = 14$

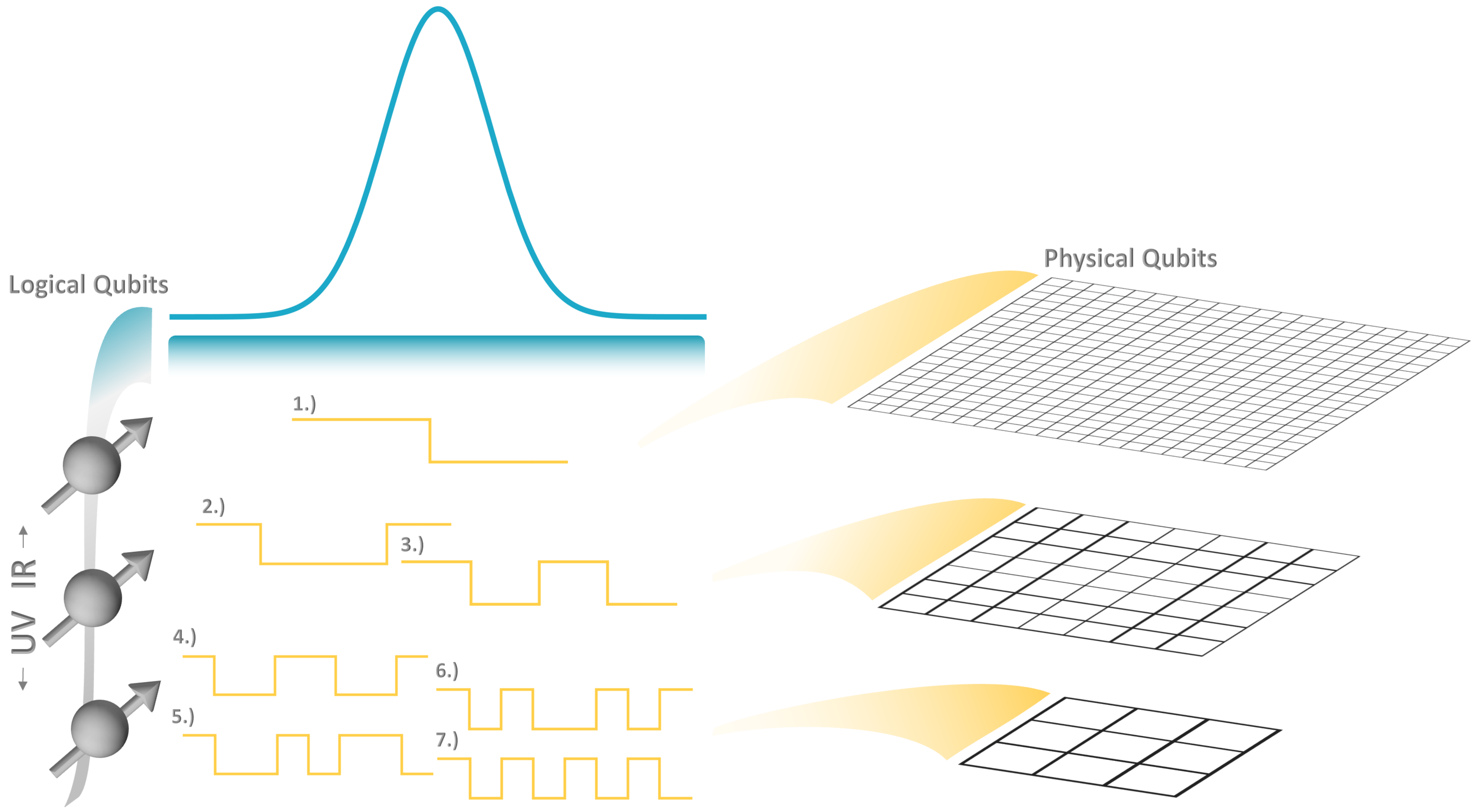
80 Pauli twirls

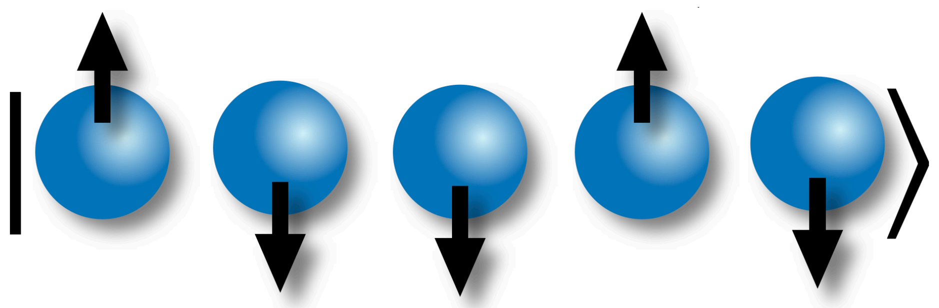
8000 shots per twirl

~ 29 % reduction in depth

~ 26 % reduction in count

Comment on Error Correction





- Hierarchically allocate physical qubits
- Dynamically evolve the allocation

N. Klco and M. J. Savage, Phys. Rev. A 104, 062425 (2021), arXiv:2109.01953 [quant-ph]

Summary

- Scalar field theory
- Adiabatic state preparation
- Error mitigation technique: Decoherence Renormalization
- Sequency Hierarchy Truncation reduces quantum resources needed.
- Qubit hierarchy and error correction

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Sequency Hierarchy Truncation (SeqHT) for Adiabatic State Preparation and Time Evolution in Quantum Simulations

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InQubator for Quantum Simulation (IQuS), Department of Physics, University of Washington, Seattle, WA 98195

(Dated: July 22, 2024)

We introduce the Sequency Hierarchy Truncation (SeqHT) scheme for reducing the resources required for state preparation and time evolution in quantum simulations, based upon a truncation in sequency. For the $\lambda\phi^4$ interaction in scalar field theory, or any interaction with a polynomial expansion, upper bounds on the contributions of operators of a given sequency are derived. For the systems we have examined, observables computed in sequency-truncated wavefunctions, including quantum correlations as measured by magic, are found to step-wise converge to their exact values with increasing cutoff sequency. The utility of SeqHT is demonstrated in the adiabatic state preparation of the $\lambda\phi^4$ anharmonic oscillator ground state using IBM's quantum computer `ibm_sherbrooke`. Using SeqHT, the depth of the required quantum circuits is reduced by $\sim 30\%$, leading to significantly improved determinations of observables in the quantum simulations. More generally, SeqHT is expected to lead to a reduction in required resources for quantum simulations of systems with a hierarchy of length scales.

Z. Li, D. M. Grabowska, and M. J. Savage, (2024), arXiv:2407.13835 [quant-ph].